(v) REFERENCES

(4) REFERENCES TO PRACTICAL RESISTORS


A2. American R.M.A. Standards (see Chapter 38 Sect. 30 and 31).


A5. G.W.O.E. "Behaviour of high resistances at high frequencies" W.W. 12.141 (June 1945) 291.


A15. American J.N.S. R.11 Specification with Amendment No. 3 (see Chapter 38 Sect. 3 for details).

A16. Canadian Standard and reports from resistor manufacturers.


(5) REFERENCES TO PRACTICAL CONDUCTORS (books)


OTHER REFERENCES


B7. American and English Standards—see Chapter 38 Sect. 3.


B12. IEEE's Standards and Data.


Additional references will be found in the Supplement commencing on page 1475.


### (ii) Impedance Calculations—Single Load

For an ideal transformer with a centre-tapped primary as shown in Fig. 5.4, the transformed load measured across the whole primary (between terminals, \(P, P\)) is equal to the transformed value \(R_t\). If however only one half of the primary is used (between either of terminals \(P\) and terminal \(C, T\)), the transformed load presented is \(\frac{1}{2} R_t\).

\[
\begin{align*}
E_1 & = N_1 \\
E_2 & = N_2 \\
R_1 & = (\frac{N_1}{N_2})^2 R_2 \\
R_2 & = (\frac{E_2}{E_1})^2 R_1
\end{align*}
\]

**Fig. 5.4. Ideal transformer with primary centre-tap and loaded secondary.**

As a practical example of the primary centre tap, consider the use of the transformer of Fig. 5.4 to feed a 500 ohm line (\(R_t = 500\) ohms). If the transformer has an impedance ratio \((N_1/N_2)^2\) equal to 10 : 1, the transformed load across the whole of the primary, e.g., when the primary is fed by push-pull amplifier, is 10 \(\times\) 500 = 5000 ohms. If, however, only one half of the primary were used for connection to a single-ended amplifier, the load presented to the amplifier would be 1250 ohms.

For an ideal transformer with a winding tapped for load matching, as shown in Fig. 5.5, the calculation of the tap to be selected for any particular load follows from the application of eqn. (1).

\[
R_1 = (\frac{N_1}{N_{AB}})^2 R_{AB} \quad \text{or} \quad R_2 = (\frac{N_1}{N_{BD}})^2 R_{BD}
\]

**Fig. 5.5. Ideal transformer with secondary tapped for load matching.**

In these equations a load connected across terminals \(A\) and \(B\) is denoted by \(R_{AB}\) and the number of turns in the secondary between these two terminals is given by \(N_{AB}\), and corresponding designations apply for any pair of terminals across which the load is connected.

If, for example, the terminals \(A, B\) provide a match with a 10 ohm secondary load with a total of \(N_{AB}\) secondary turns, the number of turns \(N_{AD}\) required between terminals \(A\) and \(D\) to provide a similar match to a 500 ohm line is given simply by

\[
\frac{N_{AD}}{N_{AB}} = 500/10 = 50, \quad \text{so that} \quad N_{AD}/N_{AB} = \sqrt{50} = 7.07.
\]

The number of turns in the 10 ohm winding is approximately one-seventh of the number of turns for matching to 500 ohms. It is, of course, permissible to use any pair of secondary terminals such as \(B, C\) or \(C, D\) and so on, so that a wide range of transformation ratios is available from a transformer arrangement such as that shown in Fig. 5.5.

### (iii) Impedance Calculations—Multiple Loads

Where two or more loads are connected simultaneously to the windings of a transformer, the conditions for matching may be determined readily by the following method.

---

*Auto-transformers* may be used with economy in some cases: a single winding is tapped to give the required turn ratio, which may be greater or less than unity, between primary and secondary. An ideal step-up auto-transformer is shown in Fig. 5.2.

**Fig. 5.2. Ideal step-up auto-transformer.**

An auto-transformer is always more economical to construct than a two winding transformer as indicated in the following tabulation:

<table>
<thead>
<tr>
<th>Type of Winding</th>
<th>Ratios</th>
<th>Total Volt-ampere rating of windings</th>
<th>Transformer Output V.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto</td>
<td>1:3 or 9:10</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Auto</td>
<td>2:1 or 1:2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Auto</td>
<td>3:1 or 1:3</td>
<td>133</td>
<td>100</td>
</tr>
<tr>
<td>Auto</td>
<td>5:1 or 1:5</td>
<td>160</td>
<td>100</td>
</tr>
<tr>
<td>Auto</td>
<td>12:1 or 1:10</td>
<td>180</td>
<td>100</td>
</tr>
<tr>
<td>Double</td>
<td>any ratio</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

The currents in the primary and secondary sections are exactly 180° out of phase, and the resistance current flowing through the common portion of the winding is the difference between the two. Where the ratio is 1:2 or 2:1, the currents in the two sections of the winding are equal.

In ideal transformer theory there is no distinction between auto- and two-winding transformers and they need not therefore be considered separately in this section.
Consider an ideal transformer having two secondaries of \( N_s \) and \( N_t \) turns connected to loads \( R_s \) and \( R_t \) respectively as shown in Fig. 5.6. It follows immediately from eqn. (2) that
\[
\frac{E_1}{E_s} = \frac{N_s}{N_t}, \quad \frac{E_2}{E_t} = \frac{N_s}{N_t}, \quad \frac{E_3}{E_3} = \frac{N_s}{N_t}
\]
and, since the transformer is an ideal one, these relations hold irrespective of the relative values of the loads.

It is sometimes convenient to draw the diagram as in Fig. 5.7, which is equivalent in every way to Fig. 5.6, E, as a special case, \( N_s/N_t = R_s/R_t \); then the voltages of the two points A and B will be the same, and the link AB may be omitted without any effect. The currents in both sections of the transformer and in \( R_s \) and \( R_t \) will then be the same.

The value of the reflected or transformed load \( R_t \) measured across the primary terminals can be obtained by considering the power in relation to the voltages specified in eqn. (3).

Let \( W_t = \frac{E^2_s}{R_t} \)
\( W_s = \frac{E^2_s}{R_s} \)
\( W_t = \frac{E^2_t}{R_t} \)

Then \( W_t = W_s + W_t \) (since there are no transformer losses)
so that \( E_t = E_t \frac{E_t}{R_t} = E_t \frac{E_s}{R_s} \)

We therefore have
\[
R = R_s \left( \frac{N_s}{N_t} \right)^2 + R_t \left( \frac{N_s}{N_t} \right)^2
\]

so that the total load \( R_t \) presented by the primary is equal to the parallel combination of the two transformed loads \( R_s (N_s/N_t)^2 \) and \( R_t (N_s/N_t)^2 \).

If in additional winding of \( N_s \) turns is connected to a load \( R_t \), we obviously have in the same way
\[
R = R_s (N_s/N_t)^2 + R_t (N_s/N_t)^2 + R_k (N_s/N_t)^2
\]

and so on, for any number of loads. Such expressions are equally applicable whether the secondary windings used are separate windings or whether they form part of a single tapped secondary. For example, if, in the transformer shown in Fig. 5.5, we have loads \( R_{1b}, R_{1c} \), and \( R_{1d} \) connected to terminals AB, AC, and BD respectively, we will have
\[
R' = R_{1b} (N_1/N_2)^2 + R_{1c} (N_1/N_2)^2 + R_{1d} (N_1/N_2)^2
\]

and so on.

A typical practical case is one in which a known power output \( W_p \) from an amplifier is fed into a known reflected impedance \( R_t \) with two secondaries feeding loads \( R_s \) and \( R_t \) (such as two loudspeakers) which are required to operate with power inputs of \( W_s \) and \( W_t \) respectively, so that \( W_p = W_s + W_t \). The required transformer turns ratio is then given by
\[
W_p = W_s + W_t \]
\( R_s \) \( W_t = W_s + W_t \) \( R_t \) \( W_p = W_s + W_t \)

so that \( R_{1s}/R_{1t} = E_s/E_t = N_s/N_t \)

and \( R_{2s}/R_{1t} = E_1/E_s = N_s/N_t \)

For example if \( W_p = 3 \) watts \( R_s = 500 \) ohms
\( W_s = 4 \) watts \( R_t = 600 \) ohms

we have
\( N_s^2 = \frac{500 \times 3}{600 \times 7} = 32.7 \)  
Therefore \( N_s = 1 \)
\( N_t = 5.3 \)

and similarly \( N_s/N_t = 4.5 \).

Expressions such as (6) may be written in the more general form
\[
N_a/N_t = \sqrt{N_s/N_t} \times \frac{W_p/W_s}{W_t/W_s}
\]

where \( N_a \) = number of turns on secondary \( n_s \)
\( N_t \) = number of turns on primary,
\( R_s \) = load applied to secondary \( n_s \)
\( R_t \) = transformed total primary load,
\( W_s \) = watts in load \( R_s \)
\( W_t \) = total watts input to primary,

but it must be noted that these relations hold only when all loads are connected so that the specified input conditions to the primary do exist. Eqn. (7) is also applicable to determine the turns ratios of two or more separate transformers feeding two or more loads from a common amplifier which delivers \( W_s \) watts into a total load of \( R_s \) ohms.

If a transformer is supplying power to two or more loads, such as loudspeakers, and one of these is switched out of circuit, the impedance reflected on to the primary will change due to the reduction of load on the secondary. In order to avoid the resultant mismatching it is advisable to switch in a resistive load, having a resistance equal to the nominal (400 c/s) impedance of the loudspeaker, so as to take the place of the loudspeaker which has been cut out of circuit. In this case the resistance should be capable of dissipating the full maximum power input to the loudspeaker. Such an arrangement will also have the result that the volume from the remaining speakers will be unaffected.

Alternatively if it is desired to switch off one loudspeaker and to apply the whole power output to a single speaker, it will be necessary to change the number of secondary turns so as to give correct matching. This change may generally be arranged quite satisfactorily by the use of a tapped secondary winding. In this case the loudspeaker would be used on the intermediate tap when both speakers are in use, and on the whole winding for single speaker operation.

It does not matter whether two or more secondary windings or a single tapped winding is employed. The arrangement shown in Fig. 5.8 is effectively identical with that of Fig. 5.6.
SECTION 2: PRACTICAL TRANSFORMERS

(i) General considerations
(ii) Effects of losses.

(i) General considerations

The treatment in section 1 based on ideal transformer theory is an extremely useful first approximation in design problems, particularly if the transformers to be used are so liberally designed that their general characteristics approximate to the ideal.

In practice it is usually necessary to take into account:
(a) The resistance of each winding
(b) The core loss
(c) The inductances of the windings
(d) The leakage inductances
(e) The capacitances between windings and between each winding and ground, and the self capacitance of each winding.

A useful equivalent circuit of a practical transformer is shown in Fig. 5.9.

Fig. 5.9. Equivalent circuit of a practical transformer.

In this equivalent circuit we have an ideal transformer with a turns ratio $N_2/N_1$ (equal to the turns ratio of the actual transformer), with the incidental characteristics of the actual transformer represented by separate reactances and resistances:

- $r_1$ = the resistance of the primary winding
- $L_1$ = the equivalent primary leakage inductance
- $r_2$ = the resistance of the secondary winding
- $L_2$ = the equivalent secondary leakage inductance
- $R_0$ = the equivalent core-loss resistance (including both hysteresis and eddy current losses)
- $L_0$ = the inductance of the primary winding
- $C_p$, $C_s$ = the primary and secondary equivalent lumped capacitances
- $R_i$ = input resistance of transformer on load
- $R_L$ = load resistance across the secondary.

Such an equivalent circuit is capable of representing a practical design with considerable accuracy, but actual calculations would be tedious and in some cases very difficult.

The reactances and resistances shown therein have varying effects on the input-output voltage ratio according to the frequency of the signal which the transformer is handling (Ref. 1). In general, the equivalent circuit can be presented in three distinct simplified forms for use when considering the transformer operating at low, medium and high frequencies respectively (Figs. 5.10A,B,C,D).

Audio transformers can be conveniently dealt with in this manner, whereas power transformers operating over a very limited frequency range can be more simply designed on the basis of Fig. 5.10A.

The whole of the equivalent circuit of Fig. 5.10A can be referred to the primary, as in Fig. 5.11 where the ideal transformer has been omitted and $r_1$ and $R_i$ multiplied by the square of the turns ratio. This is often a convenient way of making calculations.

![Fig. 5.10A. Simplified equivalent circuit for calculating the effect of losses.](image)

![Fig. 5.10B. Low frequency equivalent circuit.](image)

![Fig. 5.10C. Medium frequency equivalent circuit.](image)

![Fig. 5.10D. High frequency equivalent circuit.](image)

![Fig. 5.11. Simplified equivalent circuit referred to the primary.](image)

(ii) The effects of losses

If a transformer is designed to deliver power into a resistance load, such as valve heaters, the primary and secondary copper losses are usually designed to be of the same order.

A particular transformer reaches its maximum efficiency when the copper losses have become equal to the iron losses (proof given in Ref. A10), although this does not necessarily occur at full load unless the transformer is so designed.

The efficiency is the ratio of the output power to the input power plus the losses. For audio and power transformers for radio purposes typical efficiencies range from 70 to 95 per cent, with the majority of power transformers falling between 80 and 90 per cent.

The regulation of a transformer is defined differently in American and British practice, the primary voltage being held constant throughout.
American definition. The percentage voltage regulation is the difference between the full-load and the no-load secondary voltages, divided by the full-load voltage and multiplied by 100.

British definition. The percentage voltage regulation is the difference between the full-load and the no-load secondary voltages, divided by the no-load secondary voltage and multiplied by 100 (E.S. 205: 1943).

However, the difference between the two definitions is quite small for small percentages of voltage regulation.

The calculation of audio transformers when operating over a limited frequency range (any 200-200 c/s), and of power transformers working at 50-60 c/s, i.e. the ratio of \( R_s / E_s \) is affected mainly by the copper losses.

Thus \( E_s / I_s = \eta T \) and \( I_s / I_s \approx 1 / T \), where \( \eta \) = efficiency and \( T \) = turns ratio.

As a further consequence the impedance ratio is changed, and, making the same assumptions as above,

\[
R_s \approx r_s + \left( \frac{r_s}{T^2} + \frac{R_s}{T^3} \right)
\]

At low frequencies, the reflected impedance is altered by the shunt effect of the primary inductance, while at high frequencies, a similar change is caused by the leakage inductance and winding capacitances. This is covered in Sect. 1(ii).

The damping of a loudspeaker, connected as a load to the secondary of an output transformer, is also affected by some degree by the losses. Where an accurate indication of the damping factor is required, these losses should be taken into account. Refer to Chapter 1 Sect. 2.

### SECTION 3: AUDIO-FREQUENCY TRANSFORMERS

(i) General considerations
(ii) Core materials
(iii) Frequency response and distortion
(a) Incremental transformers
(b) Low level transformers
(c) Output transformers
(iv) Designing for low leakage inductance
(v) Winding capacitance
(vi) Tests on output transformers
(vii) Specifications for audio transformers

(i) General considerations (Reference 1)

Audio frequency transformers can be divided into three major categories,

(a) Low level input
(b) Medium level interstage
(c) Output

For design purposes it is necessary to know

(1) Operating level, usually expressed in db above or below a reference level of 1 milliwatt (i.e. 0 dbm),

(2) Frequency response, with permitted deviation, quoting reference level at which measurements are made,

(3) Permissible distortion, at specified operating levels and frequencies,

(4) Impedance, phase angle, and nature of source and load between which transformer is to be connected,

(5) Value of d.c. (if any) flowing through any winding,

(6) Hum reduction requirements,

(7) Phase shift permitted.

Operating level
This restricts the choice of core materials and determines the physical size. Suitable core alloys include MUMETAL and RADIOMETAL together with silicon steels in various grades. As MUMETAL saturates at a comparatively low flux density, it is only suitable for low level transformers. Economic factors will probably dictate which material is finally used.

### Frequency response

At low frequencies the response falls off due to the finite value of primary inductance. At high frequencies, the winding capacitance and leakage inductance are responsible for the response limitations.

### Distortion

(a) Low frequency

This is mainly dependent on the maximum operating flux density at the lowest frequency of interest. Distortion due to this cause falls off rapidly with increase in frequency. Other sources of distortion are observed when transformers operate in valve plate circuits. The drop in valve load impedance due to the shunting effect of the primary inductance may cause the valve to distort. Further, the load impedance will also change to a partially reactive one at low frequencies. The valve will therefore operate with an elliptical loadline and will introduce additional distortion unless care is taken.

A simple method of measuring the harmonic distortion in the cores of a-f transformers is described in Ref. C33.

(b) High frequency

At the end of the audio spectrum the load impedance changes again in magnitude and sign, thus causing an associated valve to generate distortion.

### Source and load

Before the design of a transformer can be proceeded with, something must be known about the impedances between which the transformer is required to operate. Assuming that it is a low level unit, for example, it may be intended to operate from a ribbon microphone, a low impedance line, a grumophone pickup or the plate of a valve. It may have to feed the grid of a pentode or triode, a line or a mixer circuit. The secondary may be shunted by a resistance or a frequency correcting network. Unless these factors concerning the external circuits are known it is not possible to predict with any degree of accuracy, the ultimate performance of the transformer.

### D.C. polarization

If an unbalanced d.c. component is present in one of the windings, this will cause a reduction in inductance over that attainable without d.c. This would require a larger transformer to meet the same performance specifications. In some cases where the d.c. magnetizing force is high, the use of high permeability alloys is not feasible. As far as practicable, unbalanced d.c. should be avoided in transformers, either by using a push-pull connection or shunt feed. For calculation of primary inductance with a d.c. component present, refer to Section 6.

### Hum reduction

When the operating level is very low, it may be found desirable to shield the transformer to decrease the hum level to a suitable magnitude. This can be achieved in several ways, MUMETAL shields up to three in number being particularly effective. An outer case of sheet metal or cast iron is normally employed. The use of a balanced structure such as a core type instead of a shell type laminations will assist in reducing the effects of extraneous a.c. fields.

#### Phase shift

In certain applications it is desirable to apply negative feedback over an amplifier incorporating one or two transformers. To achieve stability with the desired amount of gain reduction, it is necessary to exercise careful control of phase shift over a frequency range much wider than the nominal frequency range of the transformer (see Chapter 7 Sect. 3).

(ii) Core materials

High permeability alloys are now produced by several manufacturers under a variety of trade names. These are listed below, with silicon steel shown for comparison.

See also Ref. C34.
NICKEL IRON AND OTHER ALLOYS

Material
Nickelum
Permalloy C
Permalloy B
Permalloy A
Cr-Permalloy
Mo-Permalloy
Meggerm
Hiperick
45 Permalloy
Rhometal
4% Silicon Steel

\( \rho \)  
30 000  
13 000  
15 000  
12 000  
12 000  
40 000  
37  
46  
45  
95  
250  
1 200  
8 200  
300  
300  
700  
200  
200  
5  
55

\( \mu \)  
30 000  
13 000  
15 000  
12 000  
12 000  
40 000  
37  
46  
45  
95  
250  
1 200  
8 200  
300  
300  
700  
200  
200  
5  
55

\( \mu_{sat} \)  
30 000  
13 000  
15 000  
12 000  
12 000  
40 000  
37  
46  
45  
95  
250  
1 200  
8 200  
300  
300  
700  
200  
200  
5  
55

\( B-H \) Sat.
Munetal
62  
60  
55  
45  
20  
65  
55  
56  
57  
46  
45  
55  
95


Rigidity clamping is very important since there are no free laminations to vibrate under load. For this reason also, combined with the somewhat discontinuous nature of the material, the acoustic noise emitted by the block is considerably reduced, especially at higher frequencies.

PHYSICAL PROPERTIES OF CASLAM

Property
Maximum permeability
Effic. permeability
Hys. loss for \( B_{max} = 10 \text{ Kg} \) at 50-Hz
Coercive force \( B_{max} = 10 \text{ Kg} \) at 50-Hz
Sat. flux density
Total a.c. loss, \( B_{max} = 10 \text{ Kg} \) at 50-Hz
Density
Max. temperature
Resistivity (ohm-cm)
(a) Normal to plane of laminate
(b) In plane of laminate

Value
850 at 4 Kg
500 at 10 Kg
7000
2.4
18 Kg
2.5
7.0
110°C
0.04
0.003
0.03
0.002


Frequency response and distortion

Reference 3.4

Integrating transformers—Class A and B

At the mid-frequency* the amplification is very nearly equal to the amplification factor of the valve multiplied by the turns ratio of the transformer, where the secondary is unloaded.

At higher frequencies the gain falls off due to the decrease in primary resistance. The ratio of the amplification at a low frequency \( A_{lo} \) compared with that at the mid-frequency \( A_{me} \) can be expressed thus:

\[
A_{me} = \frac{1}{\sqrt{1 + \left( \frac{R_{lo}}{R_{me}} \right)^2}}
\]

where \( L_i \) is primary inductance and \( R_{lo} = R \) plate resistance plus primary resistance.

The response will fall off 3 db at a frequency such that \( oL_{lo} = R \). At a frequency such that:

\[
oL_{lo} = 2R
\]

the response will be down approximately 1 db from the mid-frequency level.

At high frequencies the leakage reactance and shunt capacitance, in conjunction with the plate and winding resistances, form a low Q series resonant circuit. Above this resonant frequency the gain will falloff rapidly. In the neighborhood of resonance the change in gain will depend on \( oL_{lo} \), the Q of the resonant circuit. This factor can be varied by adding external resistance or by winding the secondary partly with resistive wire (Ref. C3). The resonant frequency can be varied by changing the value of the total leakage inductance, \( L_i \), or the interwinding capacitances. These are both functions of the transformer structure. See also page 518.

By careful choice of core material, laminate dimensions and method of sectionalizing the winding, it is possible to achieve a frequency response extending beyond the normal audio range (Refs. C2, C4, C5, C6 and Figs. 5.12 and 5.13A).

*The mid-frequency is the frequency at which maximum gain is obtained.
As the leakage inductance \( L_s \) is proportional to the square of the turns, \( N \), it is possible to extend the high frequency response by reducing \( N \). This, of course, reduces \( L_s \) in the same proportion as \( L_p \), but this effect can be overcome by the use of a high permeability alloy.

For low level working it is usual to assume an initial a.c. permeability of 500 for silicon steel. If a MUMETAL core with an initial a.c. permeability of say, 1600, and of similar dimensions is substituted, \( N \) could be reduced in the ratio of \( \sqrt{1600/500} = 2.14/1 \) and \( L_s \), approximately 46\% over that for the silicon steel transformer. Similarly if a MUMETAL core having an initial a.c. permeability of 10 000 and of similar dimensions is substituted, \( N \) could be reduced in the ratio of \( \sqrt{10 000/500} = 53.4/1 \) and \( L_s \), approximately 28.5/1.

In those examples, the primary inductance has remained constant, but the leakage inductance has decreased over 4 times and 28 times respectively without any sectionalizing of the winding or interwinding. This indicates the improvement possible with the use of high permeability alloys.

Care must be taken with the reduction in turns, that the flux density in the core does not exceed safe limits from the point of view of distortion (see Refs. C7, C8, C9, C10).

In the case of transformers working at low levels and hence, generally, a low flux density, the distortion may not be a consideration. It is possible to achieve an improved high frequency response, while keeping the distortion at the same level at low frequencies, by increasing the core cross section and proportionately reducing the turns. This, however, increases the weight of the transformer, and also \( L_s \), so that the net reduction of \( L_s \) is not as great as was anticipated. In addition, the cost of the transformer increases, particularly if a high permeability alloy is used. Class B transformers (driver transformers) are usually called upon to handle appreciable amounts of power during the operation of the audio frequency cycle, but are designed on the basis of open circuit working when considering primary inductance. Winding resistance and leakage inductance must be kept low to avoid distortion (Refs. C21, C22).

### (iii) (b) Low Level Transformers

Transformers working about or below a zero level of 1 millivolt (1 dbm) usually employ a MUMETAL core. In most instances hum shielding is necessary to attain the desired signal to hum ratio. This is generally required to be in excess of 60 db. Satisfactory shielding can be obtained by using around the transformer one or more shields made of high permeability alloy. Where a.c. fields are strong, the outer shield is normally designed to be made of an alloy which has a high saturation flux density. Thus the inner shields of, say, MUMETAL, are working at maximum efficiency in a low a.c. field where their permeability is a maximum. Sometimes sheet steel or thick cast iron outer cases are used. More often a special alloy, such as Telsis C 2129, is used. This material has magnetic properties similar to RADIO-METAL, but is suitable for deep drawing. Inner shields are often of copper to give shielding from electric fields (Refs. C11, C12). Some improvement is possible if a core type structure is used in place of a shell type, owing to the cancellation of stray voltages induced in the winding by external fields.

As an example of the use of high permeability alloys, the following design problem is presented. Calculate the primary turns for a 50 ohm to 50 000 ohm transformer working at zero level (1 dbm). The frequency response must not fall more than 1 db below mid-frequency response, at 50 c/s. Distortion must not exceed 1\% at zero level at 50 c/s. The source impedance is 50 ohms resistive and the secondary is unloaded. Core material to be used is MUMETAL.

**1st Step.** Calculate primary inductance.

For 1 db attenuation, \( \omega L_0 = 2R \) (Eqn. 2).

\[ L_0 = \frac{2R}{\omega} = 2 \times \frac{50/2}{\pi} \times 50 = 0.32 \text{ Henry} \]

**2nd Step.** Calculate primary turns.

Assume square stack of Magnetic and Elec. Alloys No. 35 lamination. Length of magnetic path, \( l = 4.5 \) inches

Cross sectional area = 0.56 square inches.

\[ L_p = \frac{3.2A \mu l}{\pi} \]

Therefore \( N_1 = \frac{L_p}{L_s} \times \frac{I}{\mu} = \frac{3.2A \times l}{3.2A} = \frac{0.32 \times 10^4}{4.5} \]

\[ N_2 = \frac{3.2 \times 0.56 \times 10^4}{3.2A} \]

Therefore \( N = 90 \) turns.

**3rd Step.** Calculate working flux density.

Primary voltage \( E = \sqrt{W \cdot R} \)

where \( W \) = input power

Therefore \( E = \sqrt{1 \times 10^{-3} \times 50} = 0.224 \) V

and \( B = \frac{4.44 \times 50 \times 50 \times 0.56}{2000 \text{ lines per square inch}} = 310 \text{ Gauss} \)

**4th Step.** Determine percentage distortion.

Referring to Fig. 5.13B and using curve for \( \omega L_0 = 2R \), it will be noted that the percentage distortion is approximately 0.75%. This assumes that the permeability, \( \mu \), is still 10 000 at the operating flux density. In practice \( \mu \) may exceed this figure and thus the distortion as calculated above may be larger than would be measured in a finished transformer. In this problem, for simplification, no account has been taken of the stacking factor, which would modify the result slightly.

When the secondary of the transformer is loaded, \( R \) in eqn. 1 then becomes \( R_s \) as in eqns. 3 and 4 in Fig. 5.13B, and in Fig. 5.13C is read as \( R_s \). The calculation for distortion then follows in a similar manner to the unloaded secondary example worked earlier.

**(iii) (c) Output Transformers**

The factors affecting the frequency response of output transformers (Ref. C13) are similar to those affecting interstage transformers. Refer to Figs. 5.10B, C and D.
The response falls off from the mid-frequency gain by 3 db at a low frequency such that
$$\omega L_s = R_a$$
(3)

Attenuation is 1 db when
$$\omega L_s = 2R_a$$
(4)

where
$$R_a = R_p + R_w + R_L$$
$$R_p = \text{plate resistance of valve}$$
$$R_w = \text{load resistance referred to the primary}$$
$$R_L = \text{total winding resistance referred to the primary.}$$

The response falls off from the mid-frequency gain by 3 db at a high frequency such that
$$\omega L_s = R_a$$
(5)

Attenuation is 1 db when
$$\omega L_s = 0.5R_a$$
(6)

where
$$R_a = R_p + R_w + R_t$$
and
$$L_s = \text{total leakage inductance referred to primary.}$$

The gain at the mid-frequency is
$$\frac{E_o}{E_i} = \frac{R_a}{R_g}$$
(7)

where
$$E_o = \text{voltage input to grid of output valve}$$
$$E_i = \text{voltage output across } R_a$$
and
$$\mu = \text{amplification factor of output stage.}$$

It is thus possible to specify the primary and leakage inductances permissible when the frequency response requirements are known. An example will illustrate this.

Determine the minimum primary inductance and maximum leakage inductance permissible in an output transformer designed to match a pair of Class A 2A3 triode valves with a 5000 ohm load. The response is to be within 1 db from 50 to 10 000 c/s. The plate resistance of each valve is 800 ohms. Neglect $R_w$.

Make all calculations from plate to plate.

For a fall of 1 db at 50 cycles per second,
$$\omega L_s = 2R_a$$

Now
$$R_a = 500 \times 1600 = 1200 \text{ ohms approx.}$$

$$\omega L_s = 1200 \times 2 = 2400$$

$$L_s = 2400/2 \times \pi \times 10 = 7.6 \text{ henries approx.}$$

This is the value that would be measured on a bridge at low induction.

For a fall of 1 db at 10 000 cycles per second,
$$\omega L_s = 0.5R_a$$

Now
$$R_a = 5000 + 1600 = 6600 \text{ ohms.}$$

$$\omega L_s = 0.5 \times 6600 = 3300$$

$$L_s = 3300/2 \times \pi \times 10 = .52 \text{ henries approx.} = 52 \text{ millihenrys.}$$

Note particularly that distortion requirements may necessitate an increase in $L_s$ and a decrease in $R_a$.

The following table indicates the relationship between the low frequency attenuation and the ratio of $\omega L_s/R_a$:

<table>
<thead>
<tr>
<th>Loss (db)</th>
<th>Relative attenuation</th>
<th>$\omega L_s/R_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.94</td>
<td>3.63</td>
</tr>
<tr>
<td>1.0</td>
<td>0.89</td>
<td>1.94</td>
</tr>
<tr>
<td>2.0</td>
<td>0.79</td>
<td>1.30</td>
</tr>
<tr>
<td>3.0</td>
<td>0.71</td>
<td>1.00</td>
</tr>
<tr>
<td>6.0</td>
<td>0.50</td>
<td>0.58</td>
</tr>
</tbody>
</table>

5.3 (iii) OUTPUT TRANSFORMERS

The inductances required for various values of $R_a$, for a bass response loss of 1 decibel are as follows, correct to two significant figures:

<table>
<thead>
<tr>
<th>Value of $R_a$</th>
<th>Bass response down 1 db at</th>
</tr>
</thead>
<tbody>
<tr>
<td>ohms</td>
<td>150 c/s</td>
</tr>
<tr>
<td>800</td>
<td>1.7 H</td>
</tr>
<tr>
<td>1200</td>
<td>2.6 H</td>
</tr>
<tr>
<td>1500</td>
<td>3.2 H</td>
</tr>
<tr>
<td>2000</td>
<td>4.3 H</td>
</tr>
<tr>
<td>3000</td>
<td>6.4 H</td>
</tr>
<tr>
<td>4000</td>
<td>8.8 H</td>
</tr>
<tr>
<td>5000</td>
<td>11 H</td>
</tr>
<tr>
<td>7500</td>
<td>16 H</td>
</tr>
<tr>
<td>10 000</td>
<td>21 H</td>
</tr>
<tr>
<td>15 000</td>
<td>32 H</td>
</tr>
<tr>
<td>20 000</td>
<td>45 H</td>
</tr>
<tr>
<td>50 000</td>
<td>110 H</td>
</tr>
</tbody>
</table>

where $R_a$ is approximately equal to the load resistance $R_L$ in parallel with the effective plate resistance of the valve; however, see comments below. When the plate resistance is very high, as for a pentode without feedback, $R_a$ may be taken as being approximately equal to $R_L$.

For loss in bass response to be reduced to 0.5 decibel these inductances values should be increased by a factor of 1.9 times. For a reduction of 2 db the factor becomes 0.67.

Since the permeability of the core material varies with induction, the frequency response will vary with signal level—the limiting low frequency will usually extend lower as the signal level is increased. It is therefore desirable that the inductance values tabulated above be calculated or measured at low signal levels. In the case of feedback from the secondary, this effect should be taken into account.

Table 2 gives the value of inductance to provide a nearly constant output voltage, but this is only one of several requirements to be satisfied. The core distortion is a function of the ratio $\omega L_s/R_a$ (this may be derived from equations 9 and 10) and for low distortion a high ratio of inductive reactance to $R_a$ is required, and this is equivalent to having a low value of bass attenuation (see Table 1). The bass attenuation in Table 2 (1 db at specified minimum frequencies) is based on $\omega L_s/R_a = 1.94$, resulting in fairly low core distortion. Still lower core distortion would be achieved by increasing the inductance and hence, incidentally, decreasing the bass attenuation.

The other important secondary effect resulting from a finite value of inductance is the phase angle of the load presented to the output valves. If the value of inductance from Table 2 is used for bass response down 1 db at a specified frequency, the phase angle of the load presented to the output valves will be between 45° and 90° for a triode either without feedback or with negative feedback voltage. This will cause a pronounced elliptical load-line, normally resulting in severe valve distortion at this frequency, at full power output.

If $R_a$ in Table 2 is taken as being equal to $R_L$, the maximum phase angle of the load will be less than 20° for bass response 1 db down at the specified frequencies. If the factor 0.67, for bass response down 2 db, is applied to Table 2, the maximum phase angle of the load will be less than 20°. If the factor 2.0, for bass response down 0.5 db, is applied to Table 2, the maximum phase angle of the load will be about 15°. This appears to be a reasonable value for good fidelity.

Table 2 may therefore be used as a general guide to the choice of inductance values where more exact calculations are not required—see below.

Summary of general application of Table 2

To give low core distortion, nearly constant output voltage and a total load impedance effective on the valve which is not too reactive:

1. Take $R_a = R_L$.
2. Apply Table 2 as printed for ordinary use.
3. Multiply inductance values by a factor of 2 for good fidelity.
4. The specified inductances in Table 2 are to be interpreted as the minimum frequencies of operation for the transformer.
Where the source impedance is high, the high frequency fall off is determined largely by \( L_g \) and the winding capacitances as in the case of interstage transformers.

Similar devices to that employed in the construction of interstage transformers can be used to extend the range of output transformers. The use of \( \text{RAIDOTMETAL} \), specially annealed high silicon steel coil such as \( \text{SUPERSILCOR} \), and grain oriented steels, are all common in better quality output transformers (Refs. C14, C15).

It is possible to "build-in" a transformer into a half section filter and thus maintain the impedance, viewed either by the source or the load, constant over a wide frequency range. In addition the phase angle variation is reduced towards the extremes of the range. This reduces distortion and maintains full power output to a greater degree than otherwise possible.

This idea can be applied to interstage transformers and output transformers quite successfully (Refs. C19, C20). Even large modulation transformers and class B drive transformers for broadcast equipment have provided improved performance when treated in this way (Refs. C16, C17, C18).

In radio receivers and record players advantage can be taken of this "build-in" procedure, to limit the high frequency response to any given point, say 6000 c/s, with rapid attenuation thereafter. This usually involves only one extra component; a condenser across the secondary winding of the output transformer. This is quite an effective "top" limit, more so than the normal tone control. The output transformer is designed to have the necessary amount of leakage inductance for the network to function as intended.

The winding resistances are of major importance in interstage transformers, but assume greater significance in output transformers. An appreciable amount of power may be lost unless the resistances of both primary and secondary are kept to reasonable proportions. In the normal good quality transformer, the total resistance reflected into the primary side is approximately double the measured d.c. resistance of the primary. The total winding resistance (referred to the primary) will vary between 50 and 200 per cent of the load resistance, which means an insertion copper loss of 0.5 to 1 dB. This extra resistance must be considered when choosing the turns ratio to ensure the correct load (see Eqn. 2, Sect. 2), otherwise an impedance error of 10 to 20 percent will occur. Core losses will not materially affect the calculation as these losses do not reach their maximum except at full power at the lowest audio frequency of interest.

Distortion in output transformers

When a transformer has its primary connected to an audio frequency source of zero impedance, the waveform of the voltage on the secondary will be the same as that of the source—in other words there is no distortion.

When a transformer is connected in the plate circuit of a valve, the latter is equivalent to a resistance \( r_p \) in series with the source and the transformer primary. If the secondary of the transformer is also loaded by a resistance \( R_{ac} \), this is equivalent (as regards its effect on distortion) to a total primary series resistance \( R_p \), where

\[
I = \frac{1}{R_p + \frac{1}{N_1} \cdot \frac{R_p}{N_2^2}}
\]

\( N_1 = \text{primary turns} \)
\( N_2 = \text{secondary turns} \)

In the following treatment the symbol \( R_p \) is used to indicate the total effective primary series resistance, whether it is caused by \( r_p \) alone or by a combination of this and secondary loading.

The resistance \( R_p \) in series with the primary causes a voltage drop proportional to the current flowing through it, which is the magnetizing current. Now the form of the magnetizing current is far from being sine-wave, since it is distorted by the non-linear \( B-H \) characteristic of the core material.

This distorted current waveform has no bad effect when \( R_p \) is zero, but results in distortion of the voltage waveform which becomes progressively greater as \( R_p \) is increased, for any one fixed value of \( B_{max} \).

5.3 (iii) OUTPUT TRANSFORMERS

The resulting harmonic distortion with silicon steel has been calculated by Dr. N. Partridge, and his results are embodied in the formula which follows (Refs. C24, C25, C26, C27).

\[
V_{1p} = \frac{S_p}{8 \pi^2} \frac{1}{N_1^2} \cdot \frac{R_p}{(1 - 4Z_f)}
\]

(8)

This formula can be modified to include the core stacking factor, 90%, and to use inch units instead of centimeters. It then becomes

\[
V_{1p} = \frac{5.544 S_p R_p}{\pi^2} \left( \frac{1}{N_1^2} \cdot \frac{A}{(1 - 4Z_f)} \right)
\]

(9)

where

- \( S_p \) = the harmonic voltage appearing across the primary,
- \( V_{1p} \) = the fundamental voltage across the primary,
- \( R_p \) = resistance (or equivalent resistance) in series with the primary, (refer under eqn. 4, Sect. 3),
- \( A \) = cross-sectional area of core,
- \( f \) = frequency of fundamental in cycles per second,
- \( Z_f \) = impedance of primary at fundamental frequency \( \approx 2\pi f L \)
- \( L \) = inductance of primary in henrys at chosen flux density.

In most cases the final term \((1 - R_p / 4Z_f)\) can be omitted with a further simplification.

The right hand side of this equation gives the value of the fractional harmonic distortion; the percentage harmonic distortion may be obtained by multiplying this value by 100. The formula holds only for values of \( R_f / Z_f \) between 0 and 1; this limits its application to output circuits having a maximum attenuation of 3 db.

![Fig. 5.13C. The distortion coefficient of Silicon 2 as a function of B (Ref. C25).](image)

Values of \( S_p \) for Silicon 2 can be obtained from Fig. 5.13C. Similarly Fig. 5.13D can be used to determine the permeability of Silicon 2 and hence the inductance of the primary from the formula

\[
L = 2.8074A_1 \text{ heurys}
\]

(10)

where \( A_1 \) = permeability at operating flux density and core stacking factor is 90%.
5.3

then \[ R_A = \frac{2900 + 430}{10000} = 2480 \text{ ohms} \]

6th step. Calculate \[ 1 - \left( \frac{R_A}{4Z} \right) = \left( \frac{R_A}{4Z} \right) = 1 - (2480/4 \times 5000) = 0.986 \]
Thus this factor can be neglected without serious error.

7th step. Determine \( S_h \) from Fig. 5.13C
\[ S_h = 60 \]

8th step. Calculate fractional distortion
\[ V'_r = \frac{5.54 S_h R_A}{V} = \frac{5.54 \times 60 \times 75 \times 2480}{0.87 \times 1.87 \times 10^9 \times 2.5 \times 50} = 0.014 \]
Thus percentage distortion is .014 x 100 or 1.4%.

Note: High fidelity output transformers may be designed with distortion less than 0.05% at maximum power output at 50 c/s.

The value of the distortion coefficient \( S_h \) is constant for any given material operating at any one value of \( B_{max} \). The value of \( S_h \) will be different for each harmonic, but its value for the third harmonic \( S_h \) is very close to the r.m.s. sum of all harmonics \( S_{tot} \) when there is no direct current component.

It will be noticed that the curve in Fig. 5.12 reaches a minimum at about \( B_{max} = 10000 \text{ lines/sq.i.n.} \), and that the distortion coefficient rises at both lower and higher values of \( B_{max} \), although the lower rise is only slight. Actually the lower part of the curve (below the curve) drops away rapidly and eventually reaches zero at \( B_{max} = 0 \).

Some high power materials have appreciable values of the distortion coefficient even when approaching \( B_{max} = 0 \) (Ref. C9).

When there is a direct polarizing field, even as well as odd harmonic distortion are both evident. Under these conditions, below 65 000 lines/sq.i.n., the r.m.s. sum of the second and third harmonic currents \( S_{tot} \) approximates to the r.m.s. sum of all the harmonic currents.

It will be seen from this analysis that it is desirable to use a low impedance source, (e.g., triodes), and a high inductance primary for best results at low frequencies.

Distortion at high frequencies

At high frequencies distortion is produced, apart from normal valve distortion due to non-linearity of characteristics, by the leakage inductance and winding capacitances which change the magnitude and phase angle of the load impedance.

The load on the secondary, if a loudspeaker, also complicates this trouble. Here again, a low impedance source is desirable; a high impedance source will accentuate the distortion due to this effect. With Class B amplifiers, it is essential that the leakage inductance between each half of the primary be as small as possible, otherwise there will not be proper cancellation of even order harmonics, and higher order harmonics will be generated (Ref. C23). A static shield between primary and secondary will prevent any stray even harmonics being fed into the secondary by capacitive coupling. This shield will alter the winding capacitances and increase the leakage inductance, hence it must be employed judiciously.

In Class B output transformers, high leakage inductance and winding capacitances cause distortion and decrease in power output. To a considerable degree, these remarks on distortion at high frequencies also apply to Class AB transformers.

(iv) Designing for low leakage inductance*  
Assuming that the turns and winding dimensions are kept constant, leakage inductance can be progressively reduced by interweaving the winding structure until the limit is reached when

\[ \frac{a}{3N^2} < \epsilon \]  
(11)

where \( a = \) total thickness of all winding sections  
\( N = \) number of leakage flux areas  
and \( \epsilon = \) thickness of each insulation section.

*This treatment follows Crosshead, N. H. (Ref. C28).
From eqn. (11) it will be seen that the insulation between the sections is the limiting factor. Fig. 5.13E shows that the largest value of \( N^2 \) for a given number of sections is achieved when there is a half section at the end of each winding structure. Although series connections are shown, similar results can be obtained by paralleling the sections. In this case all the turns in each parallel section must be equal. An example of the use of Fig. 5.13H to determine leakage inductance follows:

A push-pull output transformer is to be wound on a former 1.875 inches long and have a winding height of 0.4 inch, allowing for clearance. The mean length of turn is 9.5 inches. The primary and secondary turns are 2700 and 120 respectively. Insulation between layers is 0.015 inch. Assuming the seventh winding arrangement in left-hand column of Fig. 5.13E, it is required to determine the leakage inductance between the whole primary and secondary. The total primary winding height is 0.24 inch and the total secondary winding height is 0.16 inch.

\[ a = 0.4 \text{ inch} \quad N^2 = 16. \]

Referring to Fig. 5.13H, intercepts of \( a \) and \( N^2 \) give 0.008.

Adding \( c = 0.015 \) gives 0.023.

---

**FIG. 5.13E**

Table of winding arrangements (Ref. C28).

**FIG. 5.13F**

Dimensions used in chart (Fig. 5.13H).

**FIG. 5.13G**

Showing use of chart (Fig. 5.13H).

**FIG. 5.13H**

Leakage inductance chart (Ref. C28).

Intercepts with the mean length of turn, 9.5 inches and winding width, \( b \), of 1.875 inches gives a vertical of 0.012.

Assuming 2700 turns, leakage inductance = 0.14 millihenry, or 14 millihenrys for 2700 turns.

**(v) Winding capacitance**

This information assists in computing the various winding capacitances of multilayer windings used in the construction of audio-frequency transformers, choke and other equipment. They apply to windings in which turns are wound on layer by layer, either interleaved or random wound, so that the P.D. between adjacent turns belonging to consecutive layers will be much greater than that between adjacent turns.

in the same layer. The capacitance effect between adjacent turns in the same layer is neglected, only that between layers being considered. Capacitance between winding and core, and electrostatic screens, if used, must also receive attention.

Fig. 5.13J illustrates a cross section of a piece of winding in which layer interleaving is used. It is seen that the dielectric between adjacent conductors in consecutive layers is complex both in shape and material. The turns or the top layer shown fall so that each turn drops in the space between two turns on the second layer, corrugating the interleaving material with a slight resulting increase in capacitance compared with that between the middle and bottom layers shown. Due to the spiral form of each layer, the position of turns in consecutive layers to one another will change at different points round the direction of winding, thus the capacitance between any pair of layers will automatically take up the average value. The composite dielectric is made up of conductor insulation, most commonly enamel, interlayering material and the triangular shaped spaces left between adjacent turns and the interleaving material. These spaces will be filled with dry air if the windings are dried out and hermetically sealed, or with impregnating compound if the windings are vacuum impregnated. The latter procedure will give rise to a somewhat higher capacitance.

![Fig. 5.13J](image)

Fig. 5.13K. Section through layer interleaved winding (Ref. C26).

![Fig. 5.13K](image)

Fig. 5.13K. Effective capacitance of a pair of layers (Ref. C29).

In practice the major controlling factor determining the total capacitance between two adjacent layers of winding is the thickness of the interleaving material, thus distributed capacitance may conveniently be estimated in terms of the thickness of interleaving material, giving the material a value of dielectric constant: empirically obtained, allowing for the average effect of the other dielectrics in the composite arrangement.

Effective layer to layer capacitance

Take a winding having only two adjacent layers, its turns distributed uniformly throughout the two layers is shown in section at Fig. 5.13K. Consider an element dx at a distance x from the end of the winding where the conductor steps up from one layer to the other. In the complete winding the elemental capacitance due to the section dx will be transformed so that it can be represented as an equivalent value across the whole winding. If the length of the whole layer is l and the capacitance per unit layer length C then the effective capacitance of the element referred to the whole winding will be \( \frac{l}{(x/l)^2Cdx} \). The capacitance due to the whole winding will be

\[
\int_0^1 \frac{l}{(x/l)^2Cdx} = \frac{l}{C}\text{.}
\]

Thus the effective capacitance of such a two-layer winding is one-third of the capacitance between two layers measured when the far ends are unconnected.

Take now a winding consisting of n whole layers: there will be \((n - 1)\) adjacent pairs of layers throughout the winding and the effective capacitance of each pair of layers, referred to the whole winding, will be a capacitance of \((2/n)^2\) times their capacitance referred to the high potential end and considered as a pair. Thus the capacitance of n whole layers referred to the whole winding becomes

\[
\frac{4}{3} \cdot \frac{2}{n^2} \cdot Cl
\]

For large values of n the capacitance becomes inversely proportional to the number of layers. Fig. 5.13J illustrates a typical winding shape together with the dimensions as used in the related diagram. The capacitance per layer, given in the foregoing formula as \(Cl\), is proportional to the product of the length per mean turn \(L_\text{m}\) and the length of layer \(L_\text{n}\).

**Vertical sectionalizing**

It is sometimes of advantage to sectionalize the winding as shown at Fig. 5.13L, each vertical space being filled completely before proceeding to fill the next one.

![Fig. 5.13L](image)

Although physically the winding will have the same overall cross-sectional dimensions its self-capacitance will be equivalent to that of a winding having \(1/N\) layer length and \(N\) layer layers, where \(N\) is the number of vertical sections. The distributed capacitance of the winding due to such sectionalizing thus is reduced by the factor \(1/N\).

Note that this rule applies only to referred interlayer capacitance and does not apply to capacitance between the top and bottom of the winding and adjacent windings or screens. The various reduction factors for vertical sectionalizing are given in Table 4.

**TABLE 4**

<table>
<thead>
<tr>
<th>Number of vertical sections</th>
<th>Distributed capacitance component</th>
<th>Winding to screen capacitance arrangement as Figure 5.13M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) One side earthy</td>
<td>(b) Centre point earthy</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>0.111</td>
<td>0.704</td>
</tr>
<tr>
<td>4</td>
<td>0.0625</td>
<td>0.6875</td>
</tr>
<tr>
<td>5</td>
<td>0.04</td>
<td>0.68</td>
</tr>
<tr>
<td>6</td>
<td>0.0278</td>
<td>0.676</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0</td>
<td>0.667</td>
</tr>
</tbody>
</table>

*The method of winding is here changed so that for the purposes of this column, the dimensions \(L_\text{m}\) and \(T_\text{m}\) will change places.

**Effect of mixing windings**

In the design of a transformer it is often necessary to mix the primary and secondary windings in order to reduce leakage inductance. This arrangement will generally be a disadvantage as regards minimizing winding capacitance, since it exposes greater surface area of winding in proximity to either the other winding or an earthed screen. If the ratio of the transformer is fairly high, then from the high impedance winding the whole of the low impedance winding appears at common audio potential, usually earthy. But if the ratio of the transformer is not very high, capacitance between
points at differing audio potentials in the two windings may have serious effects, and it is generally best to arrange the windings so as to avoid such capacitance.

Fig. 5.13N shows a cross-section suitable for an inter-valve transformer designed to operate two valves in push-pull from a single valve on the primary side. The H.T. end of the primary is earthed and is therefore diagrammatically earthed. The high potential end of the primary is adjacent to the earthy end of one of the half secondaries so that the capacitance between windings at this point is effectively from snood to earth. The two high potential ends of the secondary are remote from the primary and so minimize the possibility of unbalanced capacitance transfer from primary to one half secondary.

Fig. 5.13M. Arrangements using vertical sectionizing. See Table 1 (Ref. C29).

Fig. 5.13N. Push-pull secondary inter-valve transformer secondary arrangements (Ref. C29).

Another problem which often arises is in the design of push-pull output transformers, particularly for Class AB or Class B circuits, where it is essential that each half of the primary be well coupled to the whole secondary. From the viewpoint of leakage inductance and winding resistance, it is unimportant whether the secondary sections are connected in series or parallel. Fig. 5.13P illustrates three arrangements for a transformer of this type, each of which may be best suited under different circumstances. At (a) is an arrangement which gives minimum primary capacitance, but suffers from the defect that leakage inductance and winding resistance are unequal for the two primary halves. For Class A operation using valves requiring an optimum load of high impedance this arrangement is sometimes the best. At (b) is an arrangement intended to equalize winding resistance and leakage inductance from each half-primary to the whole secondary as well as primary self-capacitance. This arrangement is particularly suited to circuits employing low loading Class AB or Class B operation. The alternative arrangement shown at (c) results in a slightly lower referred capacitance across one half only of the primary. In general this unbalance is not desirable, but if leakage inductance is adequately high, the coupling between all the windings may be so good that the reduction in capacitance may be apparent across the whole primary.

Table 5 gives a pictorial representation of various ways in which high impedance windings may be arranged in relation to earthy points shown as snoods. The table is equally applicable if these points are earthy low impedance windings. The capacitance factors for alternative connections of the windings, with either one side or the centre point at earth potential, are given relative to the average capacitance between one end layer of the winding and one snood. The two arrangements marked with an asterisk indicate that it is necessary to reverse the direction of winding in order to achieve the capacitance factor shown.

Random winding

In what is known as random winding, no interleafing material is used. For ideal...
random winding the layers should be built up so that at all times during winding the top surface is level. Failure to do this will not greatly affect the self-capacitance, but will result in increased danger of breakdown due to electrical or mechanical stresses. As the number of layers is always large, it is convenient to reduce the calculation to simple terms of the winding dimensions.

The essential variables are: length of winding \( L_\text{m} \), length of mean turn \( L_\text{m} \), winding thickness \( T_w \) (see Fig. 5.13Q), and number of turns \( T \). Considering variation of each of these quantities in turn, the others being taken as constant: variation of \( L_\text{m} \) will vary the referred capacitance per layer as before, and additionally the effective number of layers will vary inversely as \( L_\text{m} \); variation of \( L_\text{m} \) simply varies the...
layer size, as before; variation of both \( T_w \) and \( T \) varies the number of layers in direct proportion to \( T_w^4 \) or \( T \). Thus the whole expression for variation of capacitance can be written:

\[
C_c \propto \frac{L_w L_{i,2}^{2/18}}{T_w^{1/18}}
\]

Figures 5.13T and 5.13U are based on this relation and empirical values obtained from averages results with random windings.

Example 1

A push-pull output transformer is arranged as at Fig. 5.13N: primary winding has a total of 12 layers, \( T_w = 3 \), \( k = 2 \): insulation between primary and screen, \( T_s = 15 \), \( k = 5 \); main dimensions, \( L_w = 2 \) in., \( L_{i,2} = 8 \) in. 

**Distributed capacitance**, using Figs. 5.13R and 5.13S, \( L_w = 2 \) in., \( L_{i,2} = 8 \) in., \( T_s = 3 \), \( k = 2 \) and \( n = 12 \), is \( C_{c} = 240 \mu F \). 

Capacitance from each primary to screen, substituting \( T_s = 15 \), \( k = 5 \), is \( C_a = 120 \mu F \). The capacitance factors for the arrangement of Fig. 5.13N are: (a) 0.25; (b) 0.55; (c) 0.75. Thus the total capacitance referred to the whole primary for each method of connection is:

1. \( 300 \mu F + 240 \mu F = 540 \mu F \). 
2. \( 600 \mu F + 240 \mu F = 840 \mu F \). 
3. \( 450 \mu F + 240 \mu F = 690 \mu F \).

Example 2

An intervalve transformer, to operate from push-pull to push-pull, uses a simple arrangement having both windings all in one section: \( L_w = 0.6 \) in., \( L_{i,2} = 2.5 \) in., \( T_w = 0.1 \) in., \( T \) (primary) = 4000, is \( C_a = 58 \mu F \). For secondary, \( T = 12000 \), \( C_a = 34 \mu F \).

**Capacitance coupling between one-half primary and one-half secondary**, using Figs. 5.13R and 13S, actual capacitance, \( L_w = 0.6 \) in., \( L_{i,2} = 25 \) in., \( T_s = 10 \), \( k = 3 \), is \( C_{c} = 100 \mu F \). Both windings wound in same direction, turns-factor across this capacitance referred to whole primary is \( 1 + \frac{1}{2} - 1 = 2 \), so referred capacitance is 400 \mu F. Windings wound opposite directions, turns-factor referred to primary is \( 1 + \frac{1}{2} = 1 \), so referred capacitance is reduced to 100 \mu F. Referring these two values to secondary, turns-factor are \( 1 + \frac{1}{2} = 2 \) or \( 1 + \frac{1}{2} = 1 \), giving capacitance values referred to whole secondary of 45 \mu F or 11 \mu F respectively.

A complete analysis would need to consider leakage inductance between each primary and each half secondary, and separate source and load impedances applied to each half. For this purpose, the primary and secondary shunt capacitances across each half would be 116 \mu F and 98 \mu F respectively, while the capacity coupling would be 1600 \mu F or 400 \mu F referred to half primary. The secondary shunt capacitances referred to the half primaries would be 610 \mu F.

Example 3

A direct coupled inter-valve transformer is arranged as at Fig. 5.13M: \( L_w = 1.5 \) in., \( L_{i,2} = 1 \) in., \( T_w = 20 \), \( k = 1 \) (air spaced); \( T_s = 3 \) (each winding) = 0.2 in. ; Turns 4000/12000 c.t.

**Distributed capacitance**, using Figures 5.13T and 5.13U, \( L_w = 1.5 \) in., \( L_{i,2} = 6 \) in., \( T_w = 0.2 \) in., \( T \) (primary) = 4000, gives \( C_c = 400 \mu F \). Secondary, \( T = 12000 \) gives \( C_a = 225 \mu F \) or 450 \mu F per half primary.

**Interverval capacitance**, using Figs. 5.13R and 5.13S, actual capacitance, \( L_w = 1.5 \) in., \( L_{i,2} = 6 \) in., \( T_s = 20 \), \( k = 1 \), gives \( C_{c} = 100 \mu F \). 

Vertical sectionalizing will reduce the distributed component in each case, but will also vary the interval capacitance. Using the information in Table 4 the results may be presented as in Table 5.

In practice sections for the primary and four \( \infty \) sections for each half-secondary will be the best, remembering capacitance reduction is more important in the secondary. By making the earthy end of primary and secondary at opposite ends of the vertical groups, interval capacitance coupling effects are minimized.

**TABLE 6**

<table>
<thead>
<tr>
<th>Number of vertical sections</th>
<th>Primary capacitance</th>
<th>Half secondary capacitance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distributed</td>
<td>Inter-winding</td>
</tr>
<tr>
<td>1</td>
<td>400</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>44.4</td>
<td>70.4</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>69</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>68</td>
</tr>
<tr>
<td>6</td>
<td>125</td>
<td>25.5</td>
</tr>
</tbody>
</table>

**Tests on output transformers**

Summary of R.M.A. Standard SE-106A (Sound systems)

**The distortion** shall be measured with a zero-impedance source of voltage in series with a pure resistance \( R_{eq} \) of value 0.4 (\( \pm 5% \)) times the square of the standard distribution voltage, \( V_{TR} \), from which the tap is designed to work, divided by the manufacturer's rating, \( W_{TR} \) for the power drawn by that tap at that distribution voltage:

\[
R_{eq} = \frac{0.4 V_{TR}^2}{W_{TR}} (1 \pm 0.05)
\]

Measurements shall be made at the lowest frequency of the rated frequency response or 100 c/s, whichever is the higher; at 400 c/s and at 5000 c/s if within the rated frequency response.

The power-handling capacity of a speaker matching transformer is the maximum m.m. power, drawn by the transformer at which the specified distortion (which shall be not more than 2%) is not exceeded. The power drawn by the transformer, \( W_{TR} \), shall be determined by dividing the square of the actual voltage measured across the primary terminals, \( V_{TR} \), by the square of the standard distribution line voltage, \( V_{TR} \), from which the distribution voltage. If there is more than one secondary tap, the power rating shall be given for the tap with the lowest measured power rating when properly terminated.

If the transformer is to be used in a system employing emphasized bass, a transformer must be chosen which has a rating higher than the nominal power to be handled in proportion to the bass emphasis employed. Likewise, transformers to handle organ music must have a rating at least four times the nominal power to be handled.

**The frequency response** of a speaker matching transformer is the variation of output voltage as a function of frequency, with a constant source voltage in series with a known impedance connected to the primary, expressed as a variation in dB relative to the output voltage at 400 c/s.

For measurement, the transformer shall be connected as for distortion measurement (see above). The frequency response shall be measured using a constant source voltage, which will deliver one-half rated input power, \( W_{TR} \), to the transformer at 400 c/s.
The loss of a speaker matching transformer is the inverse ratio of the power delivered by the secondary of the transformer to a pure resistance equivalent to the rated load impedance; the power delivered by the same source if the transformer is replaced by an ideal transformer of the same impedance ratio, expressed in db. For measurement, the transformer shall be connected as for distortion measurement (see above).

The impedance presented by the ideal transformer to the source shall be taken as
\[ R_T = \frac{V_{TS}}{P_{TS}} \]

The power delivered to the secondary load is \( (V_{TS}/R_{TL}) \) where \( V_{TS} \) is the voltage across the load resistance \( R_{TL} \).

The power delivered to the ideal transformer is \( (V_{T1}/R_{T1}) \) where \( V_{T1} \) is the voltage across the load resistance \( R_{T1} \).

The loss is given by
\[ R_{TS} \equiv \frac{V_{TS}}{R_{T1}} \]

The loss shall be measured at 400 c/s and at a value of source voltage at which rated power is delivered to the ideal transformer. The loss shall not exceed 2 db (equivalent to 63% minimum efficiency).

(vii) Specifications for a-f transformers

The following details are suggested for forning part of a specification for a transformer. See also Ref. C32.

In all cases, it is desirable to submit a circuit showing the transformer application when writing a specification.

(1) Input transformers

(a) Operating level; this should be quoted in db above or below specified reference level, usually 1 milliwatt.

(b) Frequency response with permissible variation in db from a reference frequency, generally 1000 c/s.

Conditions of measurement must be specified—usually normal operating conditions.

(c) Impedance ratio or turns ratio.

(d) Positions of any taps should be stated.

(e) Source and source impedance.

(f) Load and load impedance. This may be the grid of a following amplifier valve. The secondary winding may also be shunted by a frequency-correcting network; if so, full details should be given.

(g) Total r.m.s. harmonic distortion; this should be measured at max. output at the lowest frequency of interest.

(h) Minimum resonant frequency.

(i) Insertion loss in db—frequently quoted at 400 c/s.

(j) Permissible phase characteristics at lowest and highest frequencies of interest.

(k) Direct currents in windings.

(l) Magnetic and electromagnetic shielding.

(2) Interstage transformers

In general, as for input transformers, with the addition that the type of valves used in the preceding stage, together with their operating conditions, should be specified. Where push-pull input is intended, the maximum out-of-balance current should be stated.

(3) Output transformers

As (2) above. Where multiple secondary windings are employed the power to be delivered to each should be stated.

5.4 (c) Fundamental magnetic relationships

(1) Fundamental relationships

Just as we have an electrical circuit, so the core of a transformer can be regarded as a magnetic circuit through which a flux passes; its value depending on the magnetomotive force producing it and on the nature of the magnetic circuit. These are related by an equation resembling Ohm's Law for electrical circuits:

\[ F = \phi R \text{ or } F = R \phi \]

where \( F \) is the magnetomotive force (m.m.f.) in gilberts (c.g.s. electromagnetic units)

\( \phi \) is the total flux = the total number of lines of flux (or maxwells); 1 maxwell = 1 line of flux,

\( R \) is the reluctance (equivalent to resistance in electrical circuits). The c.g.s. electromagnetic unit of reluctance is the reluctance of 1 cubic centimetre of air, which is very closely that of 1 cubic centimetre of air. Reluctances are combined in series or parallel like resistances; when in series they are added.

[Diagram of a typical magnetic circuit with iron core and air gap.

Fig. 5.14. Typical magnetic circuit with iron core and air-gap.]
5.4 FUNDAMENTAL MAGNETIC RELATIONSHIPS

\[ H = F/l \]  
(5)

where \( H \) = magnetizing force in oersteds (or gausses; centimetres),
\( F \) = magnetomotive force in gausses, and
\( l \) = length of path in centimeters.

Alternatively, if \( F \) is expressed in ampere-turns, \( H \) may be expressed in ampere-turns per inch or per centimetre.

The permeability \( (\mu) \) is defined by the relationship
\[ \mu = B/H \]  
(6)

where \( B \) = flux density in gauss (maxwells per square centimetre),
\( \mu \) = permeability*,
and \( H \) = magnetizing force in oersteds.

In air, \( H \) is numerically equal to the flux density (\( B \)).

Permeability is the equivalent of conductivity in electrical circuits. Permeability in iron cores is not constant, but varies with the flux it varies. The relationship between \( B \), \( H \) and \( \mu \) is shown by the "BH characteristics" of the iron, as shown for example in Fig. 5.15. The value of \( \mu \) at any point is the value of \( B \) divided by the value of \( H \) at that point.

For example, the permeability at point C is equal to OE/OD, where OE represents the flux density at point C, and OD represents the magnetizing force at point C. The permeability is therefore the slope of the line OC. A curve may be drawn indicating the value of \( \mu \) for any value of \( H \), and this curve may be plotted on the same graph with, of course, the addition of a \( \mu \) scale (dashed curve in Fig. 5.15).

The incremental permeability is the permeability when an alternating magnetizing force is superimposed on a direct magnetizing force.

The initial permeability is the permeability at values of \( H \) approaching zero.

The hysteresis loop of a typical magnetic material is shown in Fig. 5.16. When the magnetizing force increases from zero (demagnetized condition) to the positive peak, the \( B-H \) characteristic is followed from 0 to \( A \), where \( A \) represents the maximum (peak) values of \( B \) and \( H \). As the value of \( H \) decreases to zero and then increases in the opposite direction, the path followed is along the curve ACEB, where E is the negative equivalent to A, occurring half a cycle later than A. From E, the path followed is along the curve EFGA. The area of the curve CDGFE represents power loss, known as hysteresis loss, in the magnetic material.

Point C is the value of \( B \) for zero magnetizing force (i.e., \( H = 0 \)) and it represents the residual flux density; this value of \( B \) is called the remanence, or remanent flux density.

\*Strictly speaking, \( \mu \) is measured in gausses per oersted, but it is common practice in engineering work to speak of the permeability as a pure number which is the ratio between \( \mu \) and \( \mu_0 \) where \( \mu_0 \) is the permeability of vacuum, and has a value of unity.

\*Actually this "cyclic condition" is not reached until after a number of cycles have occurred. In the early cycles the position of A falls slightly each cycle.

\*This strictly applies only to the initial cycle; the term residual flux density is used for symmetrical cyclically magnetized conditions. The latter term is also sometimes used when it is not desired to distinguish between the initial and cyclic conditions.

5.4.2 The magnetic circuit
A typical magnetic circuit is shown in Fig. 5.14. Certain assumptions are generally made for simple theoretical treatment, including:

1. That the flux confines itself entirely to the iron over the whole length of the iron path, (in practice there is always some leakage flux, which is more serious when there is an air gap).

2. That the flux is uniformly distributed over the cross-sectional area of the iron.

In Fig. 5.14 we therefore have:

Total magnetomotive force \( F \approx 1.26 \text{ NI from (2)} \)

Total reluctance \( R = R_{\text{iron}} + R_{\text{air}} \),

where \( R_{\text{iron}} = \text{reluctance of iron path} \)

and \( R_{\text{air}} = \text{reluctance of air path} \).

Total flux \( \phi = F/R = F/(R_{\text{iron}} + R_{\text{air}}) \) from (1).

The reluctance of the air gap is given by

\[ R_{\text{air}} = \frac{H}{l} = \frac{l}{A} \]

where \( l = \text{length of air gap} \)

\( A = \text{equivalent area of air gap, allowing for "ringing"} \)

\( \approx (a + b) (b + c) \)

and \( a, b = \text{actual dimensions of pole faces} \).

Values of \( k \) are given by the table below:

<table>
<thead>
<tr>
<th>( H ) oersteds</th>
<th>amperes turn/cm</th>
<th>amperes turn/inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B ) gausses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l ) centimetres</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A ) sq. centimetres</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k ) 1.000</td>
<td>0.706</td>
<td>0.335</td>
</tr>
</tbody>
</table>

The reluctance of the iron path is not constant, so that the best approach is graphical.

The magnetic potential difference \( (U) \) is the equivalent of potential difference in electrical circuits. The sum of the potential differences around any magnetic circuit is equal to the applied magnetomotive force.

Applying this to Fig. 5.14,

\[ U = U_{\text{iron}} + U_{\text{air}} \]  
(8)
where \( F \) = total magnetomotive force,
\[ U_{m} = \text{magnetic potential difference along the whole length of iron,} \]
and \( U_{a} \) = magnetic potential difference across the air gap.

This is applied in Fig. 5.17 where OM represents the applied magnetomotive force \( F \approx 1.26 NT \), and the curve shows the total flux \( \phi \) plotted against \( F \) for iron only. The shape of the curve is the same as that of the B-H characteristic of Fig. 5.15, but the vertical scale \( \phi = B \) and the horizontal scale \( F = IH \).

The "loadline" through M represents the effect of the air gap; it follows the equation
\[ U = F - \phi R_{a} \]
and its slope is \( -R_{a} \). The intersection of the "loadline" and the curve at point P gives the operating point. Therefore ON represents the magnetic potential difference along the iron path, while NM represents the magnetic potential difference across the air gap.

It will thus be seen that variation of the air gap merely changes the slope of the "loadline" ?M and moves point P, without changing the base line OM.

In order to be more generally applicable, the scales of Fig. 5.17 may be changed—vertical scale: from \( \phi \) to B (Note \( \phi = B \))
horizontal scale: from \( F \) to \( H \) (Note \( F = IH \))
as in Fig. 5.18A.

(iii) Magnetic units and conversion factors

The basic units generally adopted are the c.g.s. electromagnetic units such as the gauss, the oersted and the gilbert, with the centimetre as the unit of length. Practical units such as the lines per square inch and ampere-turns per inch are widely used in engineering design. More recently, the Giorgi M.K.S. system, with its webers and webers per square metre, has come into considerable popularity. The full range of these various systems of units is given in Chapter 38 Sect. 1.

The following table of conversion factors will be helpful in converting from one system to another.

<table>
<thead>
<tr>
<th>Magnetic Units</th>
<th>Conversion Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply</td>
<td>by</td>
</tr>
<tr>
<td>F in ampere-turns</td>
<td>0.4 = 1.257</td>
</tr>
<tr>
<td>F in gilberths</td>
<td>0.1</td>
</tr>
<tr>
<td>F in gilberths*</td>
<td>10</td>
</tr>
<tr>
<td>F in ampere-turns*</td>
<td>4 = 12.57</td>
</tr>
<tr>
<td>H in ampere-turns</td>
<td>1/0.4 = 0.796</td>
</tr>
<tr>
<td>H in oersted</td>
<td>0.4 = 2.54 = 0.465</td>
</tr>
<tr>
<td>H in maxwells/sq. in</td>
<td>1.65 = 0.155</td>
</tr>
<tr>
<td>B in gauss</td>
<td>6.45</td>
</tr>
<tr>
<td>B in maxwells/sq. cm</td>
<td>10^4</td>
</tr>
<tr>
<td>B in webers/sq. metre</td>
<td>10^4</td>
</tr>
<tr>
<td>B in maxwell/sq. in</td>
<td>10^4</td>
</tr>
<tr>
<td>B in webers/sq. metre</td>
<td>10^-8</td>
</tr>
<tr>
<td>B in webers*</td>
<td>10^8</td>
</tr>
</tbody>
</table>

SECTION 5: POWER TRANSFORMERS

(i) General

The general design principles of power transformers have been dealt with in detail elsewhere (Refs. D1, D2, D3, D4) but an outline is given below of the design procedure for small power transformers for use in radio and electronic equipment. For transformers of this kind, efficiencies ranging from 80 to 90 per cent are common. The ratio of copper to iron louser is usually about 2 to 1. Winding capacitances have little effect on circuit operation and are usually neglected. For radio receivers it is common practice to provide some form of electrostatic screening between the primary and the other windings. This can be achieved in one of several ways (Ref. D5):

(a) By using a turn of shim copper or brass of full winding pitch between primary and secondary, taking care to insulate the ends to avoid a shorted turn. This shield is then earthed.

(b) By winding the earthed low voltage winding between primary and secondary.

(c) By winding the high voltage secondary in two separate halves, one on top of the other, over the primary. The innermost and outermost leads are joined and become the centre tap which is then earthed. The two leads from the middle of the winding ther, become the high tension outers, and are connected to the rectifier plates. In this way the capacitance from primary to secondary is made very small. It is important in this method of construction to ensure that adequate insulation is used between the two plate leads within the winding, as the whole of the potential difference of the high tension winding appears between them.

With the normal method of construction the leakage inductance between each half of the secondary and the primary is unequal. In large transformers this becomes

\*M.K.S. unit: See Chapter 38 Sect. 1.
important and, to maintain balance, the primary is wound between the halves of the secondary or the latter are wound side by side over the primary.

(ii) Core material and size

Various grades of core material are available all differing in silicon content from 0.25 to 4.5 per cent (Ref. D4, 7). They feature lower loss, increasing cost and brittleness with increase in silicon. Several manufacturers make somewhat similar grades and these are listed hereunder (Ref. D8).

**Transformer Steels**

<table>
<thead>
<tr>
<th>Silicon Content*</th>
<th>M. and A.A. Baldwin</th>
<th>Sankey</th>
<th>Allegheny</th>
<th>Armco</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>Silcor 1 Quality 5</td>
<td>Super Stalloy Transf. C</td>
<td>Trancon 2</td>
<td></td>
</tr>
<tr>
<td>3.4%</td>
<td>Silcor 2 Quality 4</td>
<td>Salloy Transf. D</td>
<td>Trancon 1</td>
<td></td>
</tr>
<tr>
<td>2.4%</td>
<td>Silcor 3 Quality 3B</td>
<td>42 Quality Electrical Spec. Elec.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5%</td>
<td>Silcor 4 Quality 1</td>
<td>Loby's Armature Armature</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*This applies exactly to Silcor (Magnetic and Elec. Alloys Ltd.).

The core losses for the various Silcor grades measured at 50 c/s, with 0.014 inch sheet, at two different values of flux density are shown below.

<table>
<thead>
<tr>
<th>Watts lost per pound</th>
<th>Bmax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silcor 1</td>
<td>1.04</td>
</tr>
<tr>
<td>Silcor 2</td>
<td>0.91</td>
</tr>
<tr>
<td>Silcor 3</td>
<td>0.94</td>
</tr>
<tr>
<td>Silcor 4</td>
<td>0.99</td>
</tr>
</tbody>
</table>

It will be observed that small changes in flux density the core loss varies as the square of the flux density. For radio power transformers, work, core materials similar to Silcor 2 are commonly used. Measurements of losses can be made with a low power factor wattmeter, or by the three ammeter method. A system suitable for mass production testing has been described recently (Ref. D9).

Cold rolled, grain oriented 2.7 per cent silicon steels are becoming of increasing importance. Typical trade names for this material are “Hipersil” and “Crysralloy” (Ref. 10). Flux densities in excess of 110,000 lines per square inch (17 kilogauss), can be employed without high core loss. Owing to low core losses at such high flux densities, the application of this material to small transformers for electronic equipment is increasing. It can be used in strip form, being wound around the winding in one method of assembly. In another method, the strip core is sawn in half and the two halves clamped together around the winding. Either method results in a considerable saving in material and labour over the present method of punching laminations and hand stacking the winding with them.

The most popular laminations in current use are the E and I "scrapless" variety (Refs. D11, 12, 13, 14). These are so dimensioned that the I is punched from the window of the E thus avoiding wastage. The usual ratio of dimensions of these laminations is as follows: Window height 1, tongue 2, window width 3, length 6, magnetic path 2.

Similar laminations are styled by several different firms:

**Standard Lamination Sizes**

<table>
<thead>
<tr>
<th>Tongue</th>
<th>M.E.A.</th>
<th>Baldwins</th>
<th>Sankey</th>
<th>Allegheny</th>
<th>Chicago</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/16 in</td>
<td>18</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>EI-58</td>
</tr>
<tr>
<td>14</td>
<td>145</td>
<td>392</td>
<td>—</td>
<td>—</td>
<td>EI-625</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>217</td>
<td>70</td>
<td>70</td>
<td>EI-75</td>
</tr>
<tr>
<td>1</td>
<td>147</td>
<td>—</td>
<td>111</td>
<td>EI-11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>433</td>
<td>111</td>
<td>EI-12</td>
<td></td>
</tr>
<tr>
<td>1½</td>
<td>78</td>
<td>423</td>
<td>133</td>
<td>EI-125</td>
<td></td>
</tr>
<tr>
<td>1¾</td>
<td>152</td>
<td>362</td>
<td>—</td>
<td>EI-138</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>—</td>
<td>149</td>
<td>EI-15</td>
<td></td>
</tr>
</tbody>
</table>

(iii) Primary and secondary turns

The primary turns required can be determined from the fundamental transformer equation.

\[ N = \frac{E}{4.44 f B A} \]

where \( N \) = primary turns, \( f \) = frequency in c/s, \( B \) = max. flux density and \( A \) = cross-sectional area of core in sq. ins.

Assuming an average stacking factor of 0.90 per cent, the factor 4.44 becomes 4 in the momentum. The stacking factor reduces the apparent height of the stack, the reduction being caused by insulation, scale and burr due to die wear. With very high laminations, the stacking factor decreases to 80 per cent approximately. For a flux density of 64,000 lines per square inch (10 kilogauss) equiv. (2) becomes:

\[ N = \frac{E}{A} \]

\[ N = \frac{E}{A} \]

where \( N \) = gross cross-sectional area of core in sq. ins.

The flux density employed depends on the application, the power rating, the core material and the frequency. For oscillographs and pre-amplifiers, densities of 40,000 to 50,000 lines per square inch (about 7,000 gauss) are used. For small transformers below about 100 watts, densities up to 90,000 lines per square inch (14,000 gauss) are used, gradually decreasing to about 65,000 lines (10,000 gauss) as the transformer size increases to several hundred watts.

Havina chosen a suitable flux density, the turns required for each secondary winding may be calculated.

It can be shown that, approximately,

\[ N_t = E_t \frac{E_i}{E_i \sqrt{\eta}} \]  

where \( \eta \) = transformer efficiency.

As a first approximation it may be assumed that \( \eta = 0.85 \) and \( \sqrt{\eta} = 0.92 \).

The values thus obtained for the secondary turns may be checked by more detailed calculations. After the wire gauges have been chosen and winding resistances calculated.

For this purpose the equivalent circuit of Figure 5.12 may be used.

A secondary is to feed a rectifier and the d.c. output of the valve is specified, reference should be made to Chapter 30 or a valve data book to determine the required
secondary voltage. Allowance must be made for any voltage drops due to the d.c.
resistance of the rectifier filter.

(iv) Currents in windings

To enable the wire gauges to be chosen and to assess the copper losses in the wind-
ings, it is necessary to estimate the current in each winding. Where windings are
used to supply valve heaters and resistance loads, the winding current is the same as
the load current. In a secondary winding feeding a rectifier, the winding current
must be estimated from a knowledge of the type of rectifier and its associated filter
and their characteristics. For normally loaded full-wave rectifiers the following values
of secondary current may be used as a fairly close guide for design purposes.

Canscer input filter: The r.m.s. current in each half of the transformer
secondary may be taken approximately as 1.1 times the direct current to the load.
For further details see Chapter 30 Sect. 2.

Coke inpu Filter: The r.m.s. current in each half of the transformer secondary
may be taken approximately as 0.75 times the direct current to the load. For
further information, see Chapter 30, Sect. 4 (also Sect. 3). With half-wave rectification
there will be a d.c. component of the current which will affect the transformer design
if the total d.c. amperes turns are considerable. For half-wave battery chargers using
bulb rectifiers, this must be taken into consideration when selecting core size and wire
gauges (Ref. D15). In the case of transformers supplying full-wave rectifiers, the full load primary
current can be estimated by calculating the total secondary loading in voltamps,
allowing for a minimum efficiency of 85%, as f, the first approximation. This is then the primary
input in voltamps which, when divided by the primary voltage, will give the desired
current. Where secondaries feed a resistive load, the loading is the product of the
voltage and current. Where a secondary feeds a full-wave rectifier the load is the
product of the direct current and the direct voltage output from the rectifier plus the
power lost in the rectifier. This later can be calculated, for a condenser input filter,
from data presented in Chapter 30, Sect. 2. It should be noted that indirectly heated,
close-spaced rectifiers such as the 6X4, are more efficient than types such as the 3Y3-
GT, with its heavier filament power and lower plate efficiency. This is one reason
why the former are almost exclusively used in the majority of small a.c. radio receivers.
The primary input current as calculated above should be accurate to within 10%
for small transformers.

Wire gauges and copper losses

For the usual type of radio receiver power transformer it will be safe to choose wire
gauges on the basis of 450-800 circular mils per ampere (i.e. 2880-5500 amperes per
square inch, but values up to 1000 circular mils per ampere (i.e. 1,100 amperes per
square inch) may be desirable in larger units, or if a high flux density is used with high
loss saturation steel. The latter figure is easy to remember as 1 circular mil per milliamp.
In practice, wire gauges may be chosen arbitrarily on a basis of (say) 700 circular mils
per ampere (i.e. approximately 1800 amperes per square inch), and check calculations
should then be made to see that

(a) the build of the winding (i.e. winding height) is satisfactory for the window
space available,
(b) the copper loss is not so high as to cause excessive temperature rise,
(c) the voltage regulation is satisfactory.

Wire tables in this Handbook will simplify these calculations (Chapter 38 Sect. 19).

(v) Temperature rise

This is dependent upon the cooling area, the total loss and the ratio of iron to
copper loss. To avoid deterioration of the insulating materials, it is necessary to limit the working
temperature to 105°C for Class A insulation which includes paper, cotton,
silk, varnish and wire enamel (Refs. D16, 17). The temperature rise in the windings,
as measured by the change of resistance method, will be about 10°C lower than the maximum
(maximum hot spot) temperature. Thus with an ambient temperature of 40°C
(104°F) plus a margin of 10°C for the difference between measured and hot spot
temperature, it will be seen that the maximum permissible rise is 55°C, as measured
by the change of resistance.

It is common practice to allow 10°C margin for change in line voltage, frequency,
or operation in situations with restricted ventilation. Thus 45°C is generally accepted
as the maximum permitted rise above ambient when measured by resistance change.

The temperature difference between winding and core varies between 10° and 20°C
according to the utilization of losses. This means that even with an ambient tem-
perature of 25°C (77°F), the core temperature may be 60° (140°F). This will feel
quite hot to the touch, although the internal temperature may be well under the per-
mitted maximum. Measurement of the core temperature may be made with a spirit
thermometer if good thermal contact is maintained between the core and the ther-
ometer bulb.

The winding temperature rise can be calculated by measuring the cold resistance,
$R_s$ at an observed temperature $T_s$. After a heat run at full load the hot resistance $R$
is measured and the ambient temperature $T_i$ is again measured.

Taking the temperature coefficient of resistivity ($\alpha$) of copper as 0.00393/°C, the
temperature rise $T$ is found from the formula

$$ T = \frac{R - R_s}{\alpha R_s} $$

To correct for any changes in ambient temperature during the heat run it is neces-
sary to subtract the difference $T_i - T_s$ from $T$ to find the actual rise. An example
will illustrate this.

The primary resistance of a transformer measured when cold was 30 ohms at an
ambient temperature of 20°C. After an 8 hour full load run, the resistance was 56
ohms, while the ambient temperature was then 18°C. It is required to find the
winding temperature rise.

$$ T = \frac{36 - 30}{\frac{30 \times 0.00393}{100}} = 51°C. $$

Now $T_i - T_s = (18 - 20)°C = -2°C$.


Standard methods of testing small radio receiver power transformers have been
published (Refs. D18, D22). The cooling area of a transformer for the "scratchless"
type laminations with standard ratio of dimensions is

$$ A = \frac{7.71T + 115}{T} $$

where $A$ = lamination area in square inches,
$T$ = width of tongue
and $S$ = height of stack.

For a square stack $S = T$ and $A = 18.71T^2$ (Ref. 11).

Having calculated the iron and copper losses and the total cooling area, it is possible
to estimate the temperature rise before a sample transformer is wound. The watts
loss per square inch of cooling area is first calculated. Reference to Fig. 5.18B will
then show the temperature rise to be expected with an accuracy of approximately
± 10%.

(vi) Typical design

(Refs. D19, D20, D21, D23)

Specifications:

Primary

240 V 50 c/s

Secondary (i) 6.3 V 2.6 A

Secondary (ii) 300 V 300 V r.m.s. for full wave 6X4 rectifier with condenser
input filter to deliver 60 mA d.c.

Secondary loading

Secondary (i) 6.3 V 2.6 A or 16.5 watts

D.C. output 343 V 0.06 A or 20.5 watts
Rectifier loss (see Chap. 30) 2.0 watts approx.

Total loading 39.8 watts

Primary input (assuming an efficiency of 85%) 46.0 watts.

Thus the input current is 46/246 or 0.19 A. Core cross-section: \( A = \frac{\sqrt{39.58}}{2} = 1.12 \) square inches. Choosing pattern E1-112 lamination of Silcor 2 material, a suitable stack for calculated cross section is 1.125 inches, that is, a square stack.

Primary turns:
With a flux density of 13 kilogauss (84,000 lines per square inch) and a stacking factor of 0.9,

\[ N_p = \frac{240 \times 10^4/4 \times 50 \times 84,000 \times 1.125 \times 1.25}{1190} \]

Secondary (i) turns:

\[ N_i = \frac{E_i N_s}{E_s} \sqrt{7} = \frac{300 \times 1130/240/0.85}{1530 + 153\%} \]

Turns per volt = \( N_i/E_i = \frac{1530/300}{5.1} = (6.3 \times 5.1) = 32 \)

Wire gauges:
Assume a current density of 600-700 circular mils per ampere. Referring to wire tables, a suitable primary gauge is 29 A.W.G. enam.

Turns per layer = 95. Refer Figs. 5.18C and wire tables in Chapter 38 Sect. 19.

No. of layers = 11X/95 = 12.

R.M.S. current in secondary (i) is \( 0.06 \times 1.1 = 0.066 \) A.

Suitable wire gauge is 34 A.W.G. enam.

55. Turns per layer = 167.

No. of layers = 3060/167 = 19.

Secondary (i) gauge is twin 20 A.W.G. enam.

Turns per layer of twin wire = 16.

No. of layers = 32/16 = 2.

A twin wire is used in preference to a single wire of 17 or 18 A.W.G. in order to save winding height.

Winding Build:

Primary build = No. of layers \( \times (\text{enam. wire diam.} + \text{interlayer insulation}) \)

\( = 12 \times (32.2 + 2) \times 10^{-5} \) inches

= 0.170 inch.

Secondary (i) build = \( 19 \times (5.9 + 1) \times 10^{-4} \) inch

= 0.150 inch.

Secondary (j) build = \( 2 \times (32.4 + 5) \times 10^{-4} \) inch

= 0.075 inch.

Allowing 50 mil former thickness and 10 mil insulation between windings and over outer winding, total build is \( 0.170 + 0.150 + 0.075 + 0.050 + 0.040 \) inch

= 0.485 inch.

Winding height = 0.562 inch.

Build expressed as a percentage of window height

\( = \frac{0.485/0.562 \times 100}{86.5\%} \)

In this particular design, the heater winding is wound between primary and high tension windings to serve as a static shield.

Mean length of turn calculations—see Fig. 5.18D.

\[ a = \text{Primary build plus insulation} = 0.050 + 0.010 = 0.060 \text{ inch} \]

\[ b = \text{Primary build plus insulation} = 0.170 + 0.010 = 0.180 \text{ inch} \]

\[ c = \text{Secondary (i) build plus insulation} = 0.075 + 0.010 = 0.085 \text{ inch} \]

\[ d = \text{Secondary (ii) build plus insulation} = 0.150 + 0.010 = 0.160 \text{ inch} \]

\[ 4s = 2 \times (\text{stack} + \text{luggage}) \]

\[ 4.5 \text{ inches} \]

Primary mean length of turn = 5.44 inches

Secondary (i) mean length of turn = 6.27 inches

Secondary (ii) mean length of turn = 7.04 inches.

Winding resistance:
If each value for the mean length of turn is multiplied by the number of turns in its own winding and then divided by \( \pi \), the resulting quotient will be the number of feet of wire in each winding. Referring to the wire tables, the resistance in ohms per thousand feet for any particular gauge can be found. Dividing the wire length by a thousand and multiplying by the resistance per thousand feet, will determine the winding resistance.

Primary resistance = \( 5.44 \times 1130 \times 8.18/12 \times 1000 \) = 42 ohms

Secondary (i) resistance = \( 6.27 \times 32/12 \times 12 \times 1000 \) = 0.09 ohms

Secondary (ii) resistance = \( 7.04 \times 3060/261/12 \times 1000 \) = 470 ohms

Copper loss:
Primary copper loss = \( (0.19)^2 \times 42 = 1.52 \) watts

Secondary (i) copper loss = \( (2.6)^2 \times 0.9 = 6.61 \) watts

Secondary (ii) copper loss = \( (0.066)^2 \times 476 = 2.05 \) watts

Total copper loss = 4.18 watts

Iron loss:
Loss per pound of Silcor 2 at flux density of 13 kilogauss = 1.07 watts.

Weight of core = 2.17 lbs.

Therefore total iron loss = \( (1.07 \times 2.17) = 2.32 \) watts.

Temperature rise:
Total of iron and copper loss = \( (2.32 + 4.18) = 6.5 \) watts.

Cooling area of square stack = \( 18.71 \times (1.125)^2 = 23.6 \) sq. inches.

Therefore watts lost per sq. inch = \( 6.5/23.6 = 0.275 \) watts.

From Fig. 5.18B temperature rise = 35°C.
Efficiency:

\[
\text{Efficiency} = \frac{\text{Power output}}{\text{Power output plus losses}} = \frac{39.65}{100} = 39.65\%.
\]

Regulation, calculated from British definition:

\[
\text{Primary voltage drop} = 0.19 \times 42 = 8\ V.
\]

Regulation due to resistance of primary = \(8 \times 130/240 = 3.3\%\).

**FIG. 5.16C**

**Lamination and Coil Data**

**Example:**

"74E Wire on Square 1.25" Core

<table>
<thead>
<tr>
<th>Lamination</th>
<th>Width</th>
<th>Core Leg, In.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
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</thead>
<tbody>
<tr>
<td>Width</td>
<td>0.606</td>
<td>0.625</td>
<td>0.643</td>
<td>0.662</td>
<td>0.681</td>
<td>0.700</td>
<td>0.719</td>
<td>0.738</td>
<td>0.757</td>
<td>0.776</td>
<td>0.795</td>
<td>0.814</td>
<td>0.833</td>
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</tbody>
</table>

**Window Dimensions**

|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|

**Coil Dimensions**

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</thead>
<tbody>
<tr>
<td>Weight, Lbs.</td>
<td>2.375</td>
<td>2.425</td>
<td>2.475</td>
<td>2.525</td>
<td>2.575</td>
<td>2.625</td>
<td>2.675</td>
<td>2.725</td>
<td>2.775</td>
<td>2.825</td>
<td>2.875</td>
<td>2.925</td>
<td>2.975</td>
</tr>
<tr>
<td>Insulation, 10&quot; Diameter Tube</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
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**First" Plug**

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</thead>
<tbody>
<tr>
<td>Overall, Lbs.</td>
<td>2.125</td>
<td>2.500</td>
<td>2.875</td>
<td>3.250</td>
<td>3.625</td>
<td>4.000</td>
<td>4.375</td>
<td>4.750</td>
<td>5.125</td>
<td>5.500</td>
<td>5.875</td>
<td>6.250</td>
<td>6.625</td>
</tr>
<tr>
<td>Weight, Lbs.</td>
<td>2.125</td>
<td>2.500</td>
<td>2.875</td>
<td>3.250</td>
<td>3.625</td>
<td>4.000</td>
<td>4.375</td>
<td>4.750</td>
<td>5.125</td>
<td>5.500</td>
<td>5.875</td>
<td>6.250</td>
<td>6.625</td>
</tr>
<tr>
<td>Insulation, 10&quot; Diameter Tube</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
<td>0.850</td>
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</table>

**Fig. 5.16C. Lamination and coil data (Ref. D20).**

**Fig. 5.18D.** Method of computing mean length of turn (Ref. D20).

The percentage regulation of any secondary winding is calculated by dividing the full-load voltage drop of the winding by the open-circuit winding voltage and multiplying by 100. By adding the percentage regulation of the primary winding, the overall regulation for the secondary winding under examination is found.

Secondary voltage drop = \(2.6 \times 0.09 = 0.234\ V\)

Regulation, overall (i) = \((0.234 \times 100/6.3) + 3.3 = 6.7\%\)

Secondary (i) voltage drop = \(0.066 \times 470 = 31\ V\)

Regulation, overall (ii) = \((31 \times 100/650) + 3.3 = 8.1\%\)

Full load secondary (i) voltage = \(6.35\ V\)

and full load secondary (ii) voltage = \(300\ V + 300\ V\).

This should be a satisfactory design.

As the temperature rise is low, it might be possible to effect economies by redesign. For example, a higher loss core material could be used. Alternatively the flux density could be increased by decreasing the turns or the size of the stack. A new design could also be tried on the next smaller size lamination.

**(vii) Specifications for power transformers**

The following may be incorporated into a specification for a power transformer.

(a) Input voltage and frequency; should these quantities vary, the ranges of variation must be stated.

(b) Secondary full load voltages and currents. If tapped windings are required this should be indicated. Tolerances or voltages should be stated.

(c) Regulation of secondary voltages other than for rectifier plate circuit.

(d) Power factor of each secondary load.

(e) Rectifier system, if d.c. output is required; e.g. full wave, half wave, etc.

(f) Type of rectifier valve to be used.

(g) Type of filter circuit; e.g. choke or condenser input, etc.

(h) Capacitance of input condenser, or inductance and restance of input choke.

(i) D.C. full load current and voltage at filter input.

(j) D.C. regulation.

(k) External voltages between windings, or between windings and ground. Should transformer windings be interconnected externally, this should be indicated.

(l) Static shields. Number and position of such should be indicated.

(m) Any limiting dimensions together with mounting and lead terminations.

(n) Ambient: temperature in which the transformer is to operate.

A circuit should be supplied showing intended use of transformer. In all cases it is desirable for the design of the whole rectifier system, comprising transformer, rectifier and filter system, to be carried out by the one engineer.
SECTION 6: IRON-CORED INDUCTORS


(i) General
Iron-cored inductors fall into several different categories depending upon the circuit requirements. In some applications these inductors may have to carry a.c. only, both a.c. and d.c. They may have to work over a wide range of frequencies, or at any single frequency up to the ultrasonic range. Iron-cored inductors are employed as smoothing and swamping chokes in power supplies, as equalizer elements in audio-frequency equipment, as modulation chokes and as filter elements in carrier equipment, to mention a few varied applications.

To design such an inductor it is therefore necessary to know some or all of the following specifications—

(a) inductance, or range of inductance if variable,
(b) alternating voltage across the coil,
(c) direct current through the coil,
(d) frequency of operation,
(e) maximum shunt capacitance,
(f) minimum frequency of self-resonance,
(g) minimum Q over frequency range and/or
(h) frequency of maximum Q,
(i) d.c. resistance,
(j) shielding,
(k) temperature rise,
(l) size and weight limitations,
(m) insulation requirements.

(ii) Calculations—general
The inductance \( L \) of an iron-cored coil may be calculated using the relation

\[
L = \frac{3.2 \times N^2 \times \mu \times a}{10^5 \times l}
\]

where \( N \) = number of turns
\( a \) = effective cross-sectional area of coil in square inches
\( l \) = length of magnetic circuit in inches
and \( \mu \) = effective permeability.*

The effective permeability depends on the type of steel used, on the a.c. and d.c. flux densities in the core, and on whether the core laminations are interleaved or whether there is an air gap in the magnetic circuit.

The length of the magnetic circuit is the length measured around the core at the center of cross section of each magnetic path. Referring to Fig. 5.19, the path taken will be along the centre of each leg except where there are two windows, when each path through the centre leg will be along a line one-quarter of the way across the leg. In this latter case only a single path round the window is considered in calculating the magnetic circuit length.

Eqs. (1) is properly applicable only when the cross-sectional area of the core is uniform throughout the magnetic circuit. Where the cross section is non-uniform, a conservative value of \( L \) will usually be obtained by using for \( L \) the minimum value of the cross-sectional area. For more accurate calculations in such cases reference should be made to a suitable text-book (e.g. Ref. E1).

(iii) Effective permeability
For any given sample of laminating steel the effective permeability depends primarily on the a.c. and d.c. flux densities in the core.

Irrespective of the air gaps in the magnetic circuit, the maximum a.c. flux density \( B_{max} \) in the core is determined by the cross section of the core, the number of turns in the coil, the alternating voltage across the coil and the frequency, and may be calculated directly from the relation derived from eqn. (2) in Sect. 5,

\[
B_{max} = \frac{E}{4.44\pi N a}
\]

(2)

The d.c. magnetizing force in the core depends on the total number of d.c. ampere-turns, less the number of ampere-turns absorbed in any air gap, divided by the length of the magnetic circuit in inches (see Sect. 4).

Fig. 5.20 shows the variation of effective permeability with variation of a.c. flux density and d.c. magnetizing force for typical electrical sheet steel. It will be observed that the effective permeability increases up to a maximum as the a.c. flux density increases, and then drops rapidly due to saturation of the core. For any particular value of a.c. flux density the effective permeability decreases as the d.c. magnetizing force is increased.

Where the a.c. flux in the core is much less than the d.c. flux, it is convenient to refer to the effective permeability as the incremental permeability since it depends on the characteristics of the core material in relation to small changes in the total flux. This term is therefore generally used in dealing with filter chokes and other inductance coils having a relatively large number of turns and carrying a relatively large direct current.

(iv) Design with no d.c. flux
If there is no d.c. flux in the core the calculation of inductance is straightforward because the effective permeability depends only on the number of turns, the frequency, the core cross section, and the applied voltage. This case arises when the windings of a coil or transformer do not carry direct current or when the windings carrying direct current are so arranged that the d.c. flux in the core is zero as, for example, with transformers used in balanced push pull amplifiers.

Design of a coil to obtain a required value of inductance is therefore also quite straightforward, and the following steps may be followed—

(1) Choose an available core and calculate from eqn. (1) the number of turns required to give the necessary inductance assuming a value of effective permeability of between 1 000 and 5 000 depending on a very rough estimate of the a.c. flux density in the core.
(2) Calculate the a.c. flux density for this number of turns in relation to the known frequency and voltage across the coil.
(3) Correct the estimate of the number of turns using a revised value of permeability for the calculated flux. If the first estimate was far out, repeat steps (2) and (3).

* A more precise term is "inductance ratio."
(iv) DESIGN WITH NO D.C. FLUX

5.6

(4) If the flux density is excessive (i.e., above about 50,000 lines per square inch) increase the number of turns until the flux density is satisfactory.

In case step 4 is necessary, the magnetic circuit must be broken by an air gap and the gap adjusted until the required value of inductance is obtained; but if the larger value of inductance is satisfactory or useful the air gap may be omitted.

(5) Calculate the wire gauge required, and the copper and iron losses as in the design of a power transformer Check also that the d.c. resistance of the winding is not excessive for the purpose for which the coil is to be used.

If an air gap of length \( g \) inches is used in the core, the value of inductance becomes

\[
L = \frac{3.2N^2\mu}{10^8 J} \times \frac{1}{(1/\mu + \alpha/\beta)} \tag{3}
\]

and this expression may be used for calculations. Alternatively, it will be clear that if the air-gap is large, so that \( 1/\mu \ll \alpha/\beta \), eqn. (3) approximates to

\[
L = 3.2N^2 \cdot g / 10^8 \mu \tag{4}
\]

Fig. 5.21. Experimentally determined curve showing values of \( a \) as a function of frequency for several magnetic materials (Ref. E2).

(v) Design of high \( Q \) inductors

This sub-section follows S. L. Javna (Ref. E2) and is set out in a step-by-step form for ease of working. The following table defines the symbols used by Javna in the formulae presented here:

- \( A_e \) = Effective cross-sectional area of magnetic-flux path, in sq. in.
- \( A_s \) = Gross cross-sectional area of magnetic-flux path, in sq. in.
- \( a \) = Empirical constant, see Fig. 5.21.
- \( B \) = Maximum flux density in the core, in lines per sq. in.
- \( F \) = Fraction of core-window area occupied by copper wire of coil
- \( f \) = Frequency, in cycles per second.
- \( g \) = Actual gap length, in inches.
- \( k \) = Empirical constant, see Fig. 5.22.
- \( L \) = Inductance of iron-cored coil, in henrys.
- \( l \) = Mean length of magnetic path, in inches.
- \( m \) = Mean length of a turn of the coil, in inches.
DESIGN OF HIGH Q INDUCTORS

\[ \mu_0 = \text{Incremental permeability of core with respect to air at operating frequency.} \]

\[ N = \text{Number of turns in coil.} \]

\[ \rho_m = \text{A.W.G. (B. and S.) wire gauge number of conductor.} \]

\[ R_{e, e} = \text{Apparent r.m.s. coil resistance caused by core loss, in ohms.} \]

\[ R_{e, c} = \text{Copper loss resistance, in ohms.} \]

\[ A = \text{Total lamination window area, in sq. in.} \]

\[ V = \text{Voltage across coil, in volts.} \]

\[ w = \text{Weight of core, in pounds.} \]

A typical problem will be solved to illustrate the design procedure.

A 5 henry inductor is to be designed on a one inch square stack of Allegheny pattern EI-12 audio transformer A silicon steel annealed lamination. The voltage across the coil will be 10 V a.c. at 1000 c/s. It is required to determine the turns and gauge of wire, the gap width and Q at the operating frequency. Assume a stacking factor of 0.9.

Tabulation:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( w )</th>
<th>( A_e )</th>
<th>( A )</th>
<th>( t )</th>
<th>( m )</th>
<th>( P )</th>
<th>( k )</th>
<th>( a )</th>
<th>( \mu_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0 in.</td>
<td>1.5 lb.</td>
<td>1.0 sq. in.</td>
<td>0.9 sq. in.</td>
<td>0.75 sq. in.</td>
<td>5.5 lbs. from Fig. 5.18D</td>
<td>0.5, a typical value for this lamination</td>
<td>1.5 \times 10^{-8} \text{ from Fig. 5.22}</td>
<td>1.587 from Fig. 5.21</td>
<td>found from Fig. 5.20</td>
</tr>
</tbody>
</table>

Calculation:

\[ B = \left( \frac{1.74 \times 10^7 \times 10^6 \times 1}{1 + 2} \right) = 49.6 \text{ lines/sq. in.} \]

\[ R_{e, c} = \frac{39.5 \times 10^7 \times 10^{10}}{3.44 \times 10^7 \times 10^{10}} \]

\[ R_{e, e} = \frac{4.45 \times 10^8}{s_1 B_A A_e} \]

\[ Q = \frac{R_{e, e} + R_{e, c}}{2 \pi f_L} = 36 \]

\[ N = \frac{4.45 \times 10^8}{s_1 B_A A_e} = 5150 \text{ turns.} \]

\[ n = 49.8 + 9.96 \log (12R_{e, e}/mN) = 33 \text{ A.W.G.} \]

\[ g = \frac{(1.59 N^2 A_e - L A_e)}{2 \mu_0} = 0.08 \text{ in.} \]

Coil build check:

1. Turns/layers using 33 A.W.G. exam. = 133 from winding chart, thus number of layers = 5150/133 = 39.
2. Interlayer insulation from winding data chart = 0.0015 in., thus winding build = 35 (0.0078 + 0.0015) = 0.363 in.
3. Former thickness = 0.05 in.
4. Insulation under and over winding = 0.02 in., thus total coil build = (0.363 + 0.05 + 0.02) = 0.433 in.
5. Max. build from Fig. 5.18C = 0.43 in., thus the winding should fit the window satisfactorily.

It should be noted that \( R_{e, e} \) and \( R_{e, c} \) are usually very nearly equal and where a square stack of scrapless laminations is used, \( m \) will approximate \( l \). Due to the large air gap the inductance will remain practically independent of voltage and frequency. The basic design equations may be used and applied to almost any magnetic material. The constants \( c \) and \( k \) for other magnetic materials can be obtained from graphs drawn from measurements made in accordance with the method mentioned in the

Fig. 5.22. Experimentally determined curves showing values of "k" as a function of frequency for several magnetic materials (Ref. E2).

Design with d.c. flux (Ref. E10).

When a d.c. flux is set up in the core by unbalanced direct currents in the windings, the effective permeability is decreased, as is shown in the curves of Fig. 5.20.

The amount of d.c. flux set up in the core depends on the applied d.c. magnetizing force (i.e. the total unbalanced ampere-turns in the windings), or the length of the magnetic circuit, and on the length of any air gap in the magnetic circuit.

The effective permeability depends on

(i) the d.c. flux density in the core

(ii) the a.c. flux density in the core

and

(iii) the length of air gap in the magnetic circuit.

There is an optimum air gap giving a maximum value of the effective permeability for any particular value of total d.c. magnetizing force and a.c. flux density (Ref. E11).

For very low values of a.c. flux density, the following table gives the variation of incremental permeability with c.c. magnetizing force (A.T./inch of total length of magnetic circuit) and the ratio of the length of air gap to the total length of the magnetic circuit.
In incremental permeability of High Silicon Steel
Total d.c. magnetizing force

\[
\begin{array}{c|ccccccc}
\text{Gap Ratio} & 1.0 & 2.0 & 5 & 10 & 20 & 30 \\
\text{AT/in.} & \text{AT/in.} & \text{AT/in.} & \text{AT/in.} & \text{AT/in.} & \text{AT/in.} \\
0 & 1000 & 820 & 490 & 340 & 250 & 140 \\
0.0005 & 720 & 680 & 400 & 250 & 170 & 100 \\
0.0010 & 530 & 520 & 510 & 490 & 250 & 270 \\
0.0015 & 425 & 410 & 400 & 370 & 110 \\
\end{array}
\]

Such data may be obtained using the methods given in "Magnetic Circuits and Transformers" (Ref. A5) Ch. 7, which sets out the general methods suitable for calculation of inductance with d.c. flux in the core.

The method developed by Crowhurst (Ref. E16) makes a considerable saving in time by the use of charts, particularly when a number of inductors are to be designed.

However, when the a.c. flux density is very small, as for example in many filter chokes, the method developed by Hanna may be used.

\[V = La = \text{Volume of core (cubic inches)}\]

\[n = \text{cross-sectional area of core (square inches)}\]

\[\frac{L}{I} = \text{Inductance in henries at low a.c. flux density}\]

\[I = \text{direct current through coil}\]

\[L = \text{length of magnetic gap (inches)}\]

\[a = \text{length of air gap (inches)}\]

\[\frac{v}{l} = \text{air-gap ratio}\]

\[N = \text{number of turns in coil}\]

\[P = \text{power lost in coil}\]

\[Q = \text{calculate power dissipated in watts/sq. in.}\]

\[R = \text{temperature rise of coil}\]

\[S = \text{find gap ratio from Fig. 5.23}\]

\[T = \text{calculate air gap (v/s) x L}\]

\[U = \text{number of layers}\]

\[V = \text{calculate coil building using 0.315 in. insulation}\]

\[W = \text{calculate winding building using 0.05 in. former}\]

\[X = \text{find maximum value from Fig. 5.18C to that wading will fit window space}\]

\[Y = \text{calculate mean length of turn from Fig. 5.18D}\]

\[Z = \text{calculate coil resistance}\]

\[\text{With any particular core, the highest inductance} = \text{obtained with the lowest possible number of turns, as limited by the window space available, the permissible value of d.c. resistance, and heating of the winding with the specified current.}\]

(vii) Design by Hanna’s Method

(Ref. E13)

Stricly, the method is applicable only when the core to be used is of constant cross section throughout the length of the magnetic path, but it gives a useful first estimate in cases where the cross section is not uniform.

Let \(N\) = number of turns in coil

\(L = \text{Inductance in henrys (at low a.c. flux density)}\)

\(I = \text{direct current through coil}\)

\(a = \text{length of magnetic gap (inches)}\)

\(v = \text{length of air gap (inches)}\)

\(\frac{v}{l} = \text{air-gap ratio}\)

\(n = \text{number of turns in coil}\)

\(P = \text{power lost in coil}\)

\(U = \text{calculate power dissipation in watts/sq. in.}\)

\(V = \text{calculate current in watts/sq. in.}\)

\(R = \text{temperature rise of coil}\)

\(S = \text{find gap ratio from Fig. 5.23}\)

\(T = \text{calculate air gap (v/s) x L}\)

\(U = \text{number of layers}\)

\(V = \text{calculate coil building using 0.315 in. insulation}\)

\(W = \text{calculate winding building using 0.05 in. former}\)

\(X = \text{find maximum value from Fig. 5.18C to that winding will fit window space}\)

\(Y = \text{calculate mean length of turn from Fig. 5.18D}\)

\(Z = \text{calculate coil resistance}\)

\(\text{With any particular core, the highest inductance} = \text{obtained with the lowest possible number of turns, as limited by the window space available, the permissible value of d.c. resistance, and heating of the winding with the specified current.}\)

(viii) Design of inductors for choke-input filters

(Ref. E13)

(a) The input choke

Chapter 30 Sect. 3 Eqn. 1, 2 or 3 gives the minimum value required for the inductance of the input choke of a choke-input filter for any particular values of load resistance and frequency.

The required inductance for any particular value of load resistance may be achieved by designing the choke by Hanna’s method as given above. It is only necessary to check that the value of the a.c. flux density in the core is small, so that Hanna’s method will be applicable. For a full-wave rectifier the a.c. voltage applied to the choke is of twice the supply frequency and its peak amplitude is approximately two-thirds of the direct voltage at the input to the filter.

If, however, good regulation is required over a wide range of loads, it is necessary for the inductance of the input choke to be sufficiently high at the highest value of load resistance, that is, at the minimum value of load current. Some minimum load current greater than zero must be provided because the inductance would need to be infinite to maintain regulation down to zero load.

For a wide range of load variation (say 1:10) it would be very uneconomical to design a choke to have a constant inductance at all load currents as large as that required at minimum load. The choke is therefore designed so that its inductance will vary with the direct current through it, in such a way that the inductance is sufficiently high at all values of the load current. This method is discussed fully by Dunham (Ref. E14) and Crowhurst (Ref. E19) and reference should be made to their papers for detailed design considerations.

For many purposes, the following simple design procedure is adequate.

(1) From Eqs. (1), (2) or (3) of Chapter 30 Sect. 3, determine the required values of the choke inductance at minimum and maximum d.c. load currents.

(2) Select an interleaved core and, assuming a permeability of (say) 1000, design a choke to give the inductance required at minimum d.c. load current. For this purpose use the procedure of Sect. 6(b) above but correct the assumed value of permeability using Fig. 5.30 and taking into account the number of d.c. amperes-turns.
per inch of core path length. The peak a.c. voltage, of double mains frequency, across the choke, may be taken to be about two thirds of the d.c. voltage at the input to the filter.

(3) Using Fig. 5.20 note the change in permeability \( \mu \) when maximum d.c. load current flows through the choke and check that the value of inductance under these conditions is sufficient. If it is not sufficient an increase in turns may be satisfactory and this can be checked by repeating the procedure above. If this is not satisfactory a small air gap may be necessary, but calculations then become complex.

A choke so designed is commonly referred to as a swinging choke because its inductance varies with the direct current through it.

(b) The second choke
The design of the second choke of a choke input filter is straightforward, and Hanna's method may be used after determining the required value of inductance by the methods set out in Chapter 31 Sect. 1.

(ix) Measurements
Measurements of inductances of iron-cored coils must be made under conditions similar to those under which the coils are to be used, because the value of inductance depends to a marked degree on the a.c. flux density in the core and also on any d.c. flux set up in the core by direct currents in the windings.

Bridge methods are desirable for accurate measurements of inductance but must be arranged to simulate the operating conditions of the coil being measured. Owen and Hay bridges are widely used when there is a large flux due to direct current (Refs. B15, B16).

For many purposes it is satisfactory to determine the effective inductance (or more accurately, the impedance) of a coil by measuring the current through the coil when the rated a.c. voltage is applied to it; but this method is not usually feasible when a d.c. component is also present.

![Fig. 5.24](image)

**Fig. 5.24.** Determination of the impedance of an inductor \( L \) carrying direct current.

When a d.c. flux must be produced in the core to simulate operating conditions, the circuit arrangement of Fig. 5.24 may be used to determine the a.c. impedance of the coil. A valve voltmeter is used to adjust the a.c. voltage drop across the known variable resistance \( R \) to equality with the a.c. voltage drop across the inductance \( L \). The required value of direct current through the coil is obtained by adjusting the tapping on battery \( B \) and the rheostat \( R_1 \) (or by adjusting any other variable direct current source), the current being measured by a d.c. ammeter or milliammeter. The required alternating voltage across the choke coil is then adjusted by varying the tapping on transformer \( T \), the voltage across the coil being read on the valve voltmeter; this adjustment is not usually critical. The resistance \( R \) is then varied until the same reading is obtained on the valve voltmeter when it is connected across \( R \) by operating the switch \( S \). If the valve voltmeter responds to direct voltages, it must be connected to the circuit through a blocking condenser and grid leak as shown in the diagram.

The value \( Z \) obtained for the impedance of the coil will differ from its reactance owing to coil losses, but for most purposes it will be satisfactory to assume that it is equal to the reactance. If the frequency used is \( f \), the value of inductance is given approximately by

\[
L \approx \frac{Z}{2\pi f}.
\]

or

\[
L \approx \frac{Z}{375} \text{ Henrys for } f = 50 \text{ c/s.}
\]

\[
L \approx \frac{Z}{375} \text{ Henrys for } f = 60 \text{ c/s.}
\]

where \( Z \) is numerically equal to the value of \( R \) for balance.

For choke coils carrying direct current, and operating with a high a.c. flux density, reference should be made to the method of approximate measurement given by F. E. Temas (Ref. E15) Fig. 40 and pp. 57-58.

(x) Iron-cored inductors in resonant circuits
The performance of iron-cored inductors in resonant circuits cannot be calculated mathematically owing to the immense complexity. While most work is carried out empirically, it is helpful to have a general grasp of the problem, and this may perhaps best be carried out graphically (see Ref. E17).
REFERENCES

5.7